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ABSTRACT

This project researched three questions related to the production of cognitive skills in formal elementary and secondary education. The first question dealt with the pattern of student learning over time. It was found that, generally, the elasticity of grade eight knowledge with respect to grade eight school inputs is higher than the elasticity of grade eight knowledge with respect to grade one students who, then, might benefit from increased resource allocations in the early grades. The second question examined the preferences of teachers with respect to the level and distribution of learning within the classroom and to their own geographic location. Teachers attempt to maximize the average level of learning and to minimize the variance in learning in the classroom, though they strongly prefer to maximize achievement. The third question examined the relationship between student work effort and learning. It was found that student labor is a strong determinant of student learning. A summary of the findings and their possible policy implications are presented in the body of the report with the complete studies presented in appendixes. (Author/IRT)

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FINAL REPORT

to the

National Institute of Education

on

PROJECT ON RESEARCH IN STUDENT LEARNING

Donald R. Winkler

Community and Organization Research Institute

University of California, Santa Barbara

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PROJECT FOR RESEARCH IN STUDENT LEARNING

I. Introduction

This project has researched several questions, all related to the production of cognitive skills in formal elementary and secondary education. The first and most important question examined was the pattern of student learning over time. The results of that research have implications for the optimal allocation of resources over time.

The second question researched here was the preferences of teachers with respect to (i) the level and distribution of learning within the classroom and (ii) their own geographic location.

The third question examined was the relationship between student work effort and student learning. Work effort can be directly measured or proxied by use of attitudinal variables. Our research primarily used a direct measure of work effort.

Each of the questions which we attempted to answer in this research project is discussed in more detail below. For each question we provide a summary of our findings and their possible policy implications. The complete studies which lie behind these findings and conclusions are included in the appendices to this report.

II. Student Learning Over Time

The conventional belief among psychologists and educators has been that the level of student knowledge upon reaching adulthood is maximized when school resources are concentrated in the early years. This belief, and the scanty empirical evidence which lies behind it, has provided the major rationale for early childhood education programs and for focusing compensatory education

programs in the early years of school.

We first explored this issue theoretically in a paper titled "On the Optimal Allocation of Resources in the Production of Human Capital". By postulating a Cobb-Douglas production function of knowledge and assuming a fixed school budget over time, we conclude that the optimal investment trajectory is almost always one where expenditures per pupil increase over the schooling period.

The theoretical finding is supported by our empirical research. Using longitudinal data on school inputs and scores on standardized examinations for cohorts of pupils, we estimated a production function for the level of knowledge at grade eight. Distributed lag analysis was employed to estimate the productivity of school resources over the school-life (eight years) of the child. The full study is given in "Production of Human Capital Over Time" in Appendix B.

The empirical estimation indicated that the productivity of school resources increases between grades one and eight for the sample as a whole. In general, the elasticity of grade eight knowledge with respect to grade eight school inputs is higher than the elasticity of grade eight knowledge with respect to grade one school inputs. The corresponding policy implication is that the annual level of school expenditures should increase over time in order to maximize grade eight achievement. In other words, expenditures should be higher in grade eight than grade one.

However, we find the pattern of productivity of school inputs varies between subgroups of pupils. In general, the productivity increases over time for high income and high achieving pupils, while the productivity remains relatively constant over time for low income and low achieving pupils. The corresponding policy implications are that expenditures should increase with grade level for the high income and high achieving pupils, while expenditures

should remain relatively constant over grade levels for low income and low achieving pupils.

Like most school districts in the United States, we found that per pupil expenditures generally increase with grade level for students in the school district which we studied. This pattern is satisfactory for the high income, high achieving pupils. But low income, low achieving pupils would be better off, in terms of eighth grade test scores, if this pattern were altered to give them more resources in the early years.

These findings then provide empirical support for focusing compensatory education programs and expenditures in the early years of a child's formal education. However, the findings provide no support for enriched early childhood programs for all children. This latter finding is of special importance, since there currently is considerable political pressure to provide enriched early childhood experiences to high income as well as low income children.

Our findings, while interesting, do not necessarily extend to school districts unlike the one actually studied, which is located in an urban, California city. Furthermore, our research entails several assumptions of which the policymaker should be aware. These assumptions are listed and discussed in the text of the paper itself.

III. Teacher Preferences

We originally set out to examine the preferences of teachers with respect to their geographic location. Other studies have reported that teachers have strong preferences for teaching children of high socio-economic status. A sorting mechanism appears to operate in many communities. Most teachers prefer to teach in the schools with high income or high achieving pupils. However, only the high quality teachers are selected to teach in those schools.

If the quality of the teaching faculty is a function of the achievement

level of the pupils, the estimates of teacher productivity obtained in a single equation learning model will be biased. We tested the hypothesis that teacher quality is not a function of pupil achievement, but found we could not reject it. Hence, this simultaneous equations problem was ignored in the remainder of our research.

The usual model of learning assumes that teachers attempt to maximize the average level of learning in their classroom. However, teachers may have other objectives as well. For example, they may wish to minimize disorder or disciplinary problems. Or, they may wish to maximize the learning only of the brightest children. The particular teacher objectives will determine how that teacher allocates his or her time among the children in the classroom. If the teacher objective is not simple maximization of achievement, the usual model may provide biased empirical results with faulty policy implications.

We attempted to estimate teacher preferences with respect to (i) the average level of learning in the classroom and (ii) the variance in learning in the classroom. Learning was measured by the difference in scores on Stanford Reading Achievement Tests between two consecutive grade levels. We found that teachers attempt to maximize average level of learning and minimize variance in learning. However, they strongly prefer maximizing achievement to minimizing variance. The full results are given in "Teacher Preferences With Respect to the Level and Distribution of Scholastic Achievement."

This finding does not have any obvious policy implication. However, it does have relevance for future research in learning. Teachers do not simply attempt to maximize learning. Ignoring the complexity of the teacher objective function may create misleading results and policy implications.

IV. Student Work Effort and Learning

Several studies, including the Coleman Report, have noted a strong

statistical relationship between student attitudes about the future or attitudes about their control over their environment and student learning. We, too, find such a relationship when we regress level of learning on such student attitudes.

However, these student attitudes are mainly a proxy for student work effort in the school. If the student believes he or she has no control over his environment or lives for the present instead of the future, he or she is likely to spend less time effectively engaged in learning activities than a student who believes the opposite. We find a high correlation between these student attitudes and actual student work effort for the sample of students used in our empirical work.

Our research and findings on student work effort are reported in the attached paper titled "Student Labor Supply in Learning". In the production equation we find that student labor is a strong determinant of student learning. The elasticity of learning with respect to student labor is approximately .25. In the labor supply equation we find that the productivity of student labor is a strong determinant of the amount of labor provided in learning. The elasticity of labor supply with respect to the marginal product of labor is approximately .30.

When the level of resources received by pupils in the home or the school is changed, student labor may change as well. There are two offsetting effects. The "income" effect is that an increase in, for example, school inputs results in higher learning, so the student is likely to reduce his work effort and take more leisure. The "substitution" effect is that an increase in school inputs leads to a higher marginal productivity of labor which provides an incentive to the student to increase his work effort. We find for our sample that an increase in school or home inputs in the production equation tends to increase the amount of labor provided by the student in learning. In other words, the

substitution effect exceeds the income effect.

Student work effort and student attitudes vary greatly between races and between socio-economic groups. Research on student labor supply is important in determining policies which may induce students to increase their work effort. The research done in this project in no way provides definitive policy implications but is an important first step towards estimation of more realistic student labor supply functions.

V. Future Research

The questions investigated here are very important ones which have not previously been addressed. However, the research undertaken in this project needs to be replicated and improved before strong policy implications can be made.

One possible fruitful area of research is learning over time. Interest groups are currently advocating universal enriched early childhood education on the basis of very little empirical evidence. Our research suggests early childhood education may be an inefficient use of resources for most the nation's children. However, further research obviously needs to be done in this area. Longitudinal studies of learning should receive high priority in terms of funded research. Only through longitudinal studies can one answer questions regarding the optimal allocation of resources over time.

Another area of research which deserves high priority is the study of the relationship between the amount of time spent learning and the amount of knowledge learned. In particular, it is important to know what factors affect student work effort, and what is the most cost-effective method of increasing student work effort in or out of school. Related to this is an important question: Is it more cost-effective to increase learning by increasing school resources or by changing those variables which influence student work effort?

Compensatory education programs which attempt to increase work effort may improve learning more than programs which simply provide an enriched school environment.

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APPENDIX A

ON THE OPTIMAL ALLOCATION OF RESOURCES IN THE PRODUCTION OF HUMAN CAPITAL

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1. Introduction

The educational output which is of the most economic interest is the change in the stock of human capital, broadly defined, embodied by students. Assuming schools wish to maximize this output, they face the problem of optimally allocating a constrained budget over inputs. This problem has been the focus of much economic research on educational production functions, starting with Coleman [1]. However, one aspect of this problem has been completely ignored to date: what is the optimal allocation of resources over time in terms of maximizing the stock of human capital existing at the end of the schooling period?

Should resources be concentrated in the early grades, as some psychologists (notably Bloom [2]) contend? Or should the time distribution of resources be weighted towards secondary education, as is typically observed? The answer, of course, largely depends on the dynamics of individual learning. The static micro educational production functions, as estimated, for example, by Coleman [1] are irrelevant with respect to longitudinal resource allocation considerations. Interaction in educational outputs over time are ignored, and as a result the terminal output is independent of the point in time when inputs were utilized. To deal with the optimal allocation of school inputs over time it is therefore necessary to specify the educational production process in the form of a dynamic model.

In this paper we theoretically derive the optimal path of school investment or school resource allocation over time for an individual student The characteristics of the path vary with the properties of the dynamic growth model. The basic model employed here has its intellectual origin in the economist's model of capital accumulation. The analogy is not a new one; Ben Porath [3] has used a similar model to determine the optimal

path of individual investment in human capital over time. However, our application is different in that the decision-maker is the school, not the individual, and the objective is not maximization of consumption over a lifetime, but maximization of the stock of human capital upon termination of secondary education.

In what follows we first develop the theoretical model. We postulate a general production function of human capital and employ the techniques of optimal control to derive a general solution to the problem. Next, we specify the dynamic growth model of human capital, which like its analogue includes a rate of depreciation or obsolescence and a production function for new capital. We derive the optimal time paths of school investment for a new capital production function of the Cobb Douglas type. The economic interpretations and policy implications of the solutions are discussed.

2. The General Human Capital Accumulation Model

The major assumption which lies behind this analysis is that schools attempt to maximize the human capital embodied by each student upon termination of secondary school. It has thereby been implied that (a) it is compulsory for students to finish high school, but (b) there is no constraint on the level of scholastic achievement attained, and (c) the school is not concerned with the distribution of achievement.

Human capital is here broadly defined as a multi-dimensional vector of traits, represented by a vector $\underset{\sim}{k}(t)$. It may include elements which produce money income, income in the form of self-produced consumption goods and psychic income. Our assumption restated is that schools attempt to maximize the probability that students attain maximum income producing capacities.

The stock of the human capital embodied in an individual student changes over the period of schooling. The change in the vector of human capital $\underline{k}(t)$ over time is considered simply as a linear function of the preceding stock and the new human capital produced, analogous to changes in physical capital, such that

$$\dot{\underline{k}} = \frac{d\underline{k}}{dt} = -A \cdot \underline{k}(t) + \underline{f}(\underline{k},\underline{u},\underline{z}) \qquad (1)$$

where the matrix A is diagonal with the rates of obsolescence or forgetting associated with each of the human capital components as elements, and the vector function $\underline{f}$ represents the production of new human capital. This vector function includes as arguments the stock of human capital, a vector of controllable inputs, $\underline{u}(t)$, and a vector of exogeneous inputs, $\underline{z}(t)$, representing home environment. We assume the home environment is known over the school period and can be expressed as a smooth function of time, so that the symbol $\underline{z}(t)$ can be replaced by time only in Eq. (1). The assumptions made concerning the production functions $\underline{f}$ and that the marginal products for $\underline{k}$ and $\underline{u}$ are non-negative and that the function $\underline{f}$ is concave.

Recent psychological research in learning provides support for a human capital accumulation model as given in Eq. (1). In particular, prior learning levels have been found to be strong determinants of changes in learning by Bloom [2] and Block [4].

The school is now faced with the task of distributing an available budget, B, over input resources and over time, so as to maximize a scalar index of the vector $\underline{k}(t)$ at the end of the learning period (T), given an initial human capital level of the student of $\underline{k}_0$ and subject to the human capital accumulation equation of Eq. (1).

3. Optimality Conditions

We employ the techniques of optimal control theory to find the optimality conditions for the general model. For simplicity we limit the dimension of the human capital vector and of the input vector to one. This simplification does not in any way reduce the generality of the solution method followed. After introducing r as the social discount rate, the variable x(t) is defined as,

$$x(t) = \int_0^{t_f} e^{-rt}\, u(t)dt \tag{2}$$

and the problem can be restated to be:

$$J = \max_{u \in U} k(T) \tag{3}$$

subject to:
$$\dot{k} = -A\, k(t) + f(k,u,t)$$
$$\dot{x} = e^{-rt}\, u(t) \tag{4}$$

given the initial time constraints:

$$k(t=0) = k_o$$
$$x(t=0) = 0 \tag{5}$$

and the terminal time constraints:

$$x(t=T) = B \tag{6}$$

with the limitations on the state and control

$$k(t) \geq 0 \tag{7}$$
$$0 \leq e^{-rt}\, u \leq (B - x(t))$$

In our application of the maximum principle we shall ignore the constraints of Eq. (7) for the time being.

Varaiya [5] has shown that the Pontryagin maximum principle can be used to establish the conditions for an optimum for a problem of the type described.

Define the Hamiltonian:

$$H = -p_1 Ak + p_1 f(k,u,t) + p_2 e^{-rt} u \tag{8}$$

where the costate variables p_1 and p_2 are defined by

$$\dot{p}_1 = -\frac{\partial H}{\partial k} = p_1 A - p_1 \frac{\partial f}{\partial k}$$

$$\dot{p}_2 = -\frac{\partial H}{\partial x} = 0 \tag{9}$$

$$\text{or } p_2(t) = \text{constant}$$

and the added transversality condition derived from the constant term of the objective function:

$$p_1(t=T) = 1 \tag{10}$$

The necessary condition for the maximization of the criterion is that the control is chosen so as to maximize the Hamiltonian at each point in time:

$$\frac{dH}{du} = 0 \tag{11}$$

The optimal control is then found from the equation:

$$\frac{dH}{du} = p_1 \frac{\partial f}{\partial u} + p_2 e^{-rt} = 0 \tag{12}$$

The differential equations in state and costate of Eqs. (4) and (9) with the transversality conditions of Eqs. (5), (6) and (10) form a two point boundary value problem, once the control is eliminated from the differential equations using Eq. (12).

From Eq. (12) and the assumption that marginal products are positive, we find

$$\text{sign } (p_1) = -\text{sign } (p_2) \tag{13}$$

Since the first costate has to be positive at the terminal time, and the second costate is constant, the first costate variable is positive over the whole planning horizon.

Hence, from Eq. (10) and (12) it follows that

$$p_1(t) > 0$$

and (14)

$$p_2 < 0$$

An economic interpretation can be given to the costate variable and the Hamiltonian introduced in the optimization analysis. The first costate variable reflects the demand price, in terms of k(T), per unit of net new additions to the stock of human capital. According to Eq. (9) the logarithm of the price decreases if the marginal productivity of human capital exceeds the constant rate of obsolescence. The second costate variable can be interpreted as the negative of the supply price, in terms of k(T), of inputs for the human capital process. This price is exogeneous to the system and determined by the budget available.

The Hamiltonian can then be interpreted as the net "profit" at time t from the net investments in human capital. The profit at a given point in time is the difference between the value of these investments and the total costs of the inputs required to produce the net new human capital.

The optimal investment trajectory can be computed once the functional form and the parameters of the new capital formation function, f, as well as the obsolescence and social discount rates are specified. Linear new capital

formation functions are the most widely used by educational economists and psychologists, notably by Winkler [6], Murnane [7] and Carroll [8]. But it can be easily shown that the solution of the human capital maximization problem with a linear human capital accumulation model is always of a bang bang type. The policy implications of the linear model, i.e. that the full budget be spent at the initial time or at the final time, are unrealistic.

As an alternative we assume a new capital formation function of the Cobb-Douglas type where human capital and school inputs are single dimensional:

$$f(k,u) = b\, k(t)^{\alpha} u(t)^{1-\alpha} \qquad (15)$$

4 The Cobb-Douglas New Capital Production Function

Now we have specified the human capital accumulation to be of the type:

$$\dot{k} = -ak + b\, k(t)^{\alpha}\, u(t)^{1-\alpha} \qquad (16)$$

where a and b are scalars.

With Eq. (12) the optimal control u^0 can be expressed in the capital accumulation and the costates:

$$u^0(t) = k(t)\left[\frac{(\alpha-1)bp_1 e^{rt}}{p_2}\right]^{\frac{1}{\alpha}} \qquad (17)$$

The two point boundary value problem of the preceding section is now described by the following four differential equations:

$$\dot{k} = k\left\{-a + b\left[\frac{(\alpha-1)bp_1 e^{rt}}{p_2}\right]^{\frac{1-\alpha}{\alpha}}\right\}$$

$$\dot{x} = e^{-rt} k\left[\frac{(\alpha-1)\, bp_1 e^{rt}}{p_2}\right]^{\frac{1}{\alpha}}$$

$$\dot{p}_1 = p_1 \{a - b\alpha[\frac{(\alpha-1)bp_1 e^{rt}}{p_2}] \frac{1-\alpha}{\alpha}\}$$

if $p_1 = 0$, p_2 has definite value in terms of p_2; this is true anywhere $p_1 = 1$.

$\dot{p}_2 = 0$ or p_2 is constant

with the transversality conditions:

$$\begin{aligned} k(0) &= k_0 \\ x(0) &= 0 \\ x(T) &= B \\ p_1(T) &= 1 \end{aligned} \qquad (19)$$

provided the constraints

$$\begin{aligned} k &\geq 0 \\ 0 \leq u\, e^{-rt} &\leq (B - x(t)) \end{aligned} \qquad (20)$$

are satisfied.

The general direction of the optimal trajectories of human capital accumulation and costates can be found from a phase diagram (see Figure 1) for k and p_1 in the third quadrant of the $p_1 - p_2$ plane (where $p_1 > 0$ and $p_2 < 0$).

[Figure 1 here]

The lines $p_1(t)$ and $\dot{p}_1 = 0$ (or $p_1(t) = [\frac{a}{b}]^{\frac{\alpha}{1-\alpha}} \frac{p_2}{b(\alpha-1)} e^{-rt}$) determine the two general areas where feasible p_1 and k trajectories can be found, i.e.

$$(1)\ \begin{cases} \dot{p}_1 < 0 \\ p_1(t) > 1 \end{cases} \quad \text{and} \quad (2)\ \begin{cases} \dot{p}_1 > 0 \\ p_1(t) < 1 \end{cases} \qquad (21)$$

The first area (A) coincides with values of p_2 corresponding to large budgets

such that $0 > p_2 > [\frac{ab}{a}]^{\frac{a}{1-a}} b(a-1)\, e^{rt}$. In this whole area the capital stock is increasing. In the second feasible area we find a triangle where the capital stock increases (C) and an area where it decreases (B). These three areas, which are drawn in Figure 1 for r = 0, are more precisely defined:

$$\left.\begin{matrix}\dot{k} > 0 \\ \dot{p}_1 < 0\end{matrix}\right\} \quad p_1(t) > [\frac{a}{ab}]^{\frac{a}{1-a}} \frac{p_2}{b(a-1)}\, e^{-rt} \tag{22}$$

$$B: \left\{\begin{matrix}\dot{k} < 0 \\ \dot{p}_1 > 0\end{matrix}\right\} \quad 0 < p_1(t) < [\frac{a}{b}]^{\frac{a}{1-a}} \frac{p_2}{b(a-1)}\, e^{-rt} \tag{23}$$

$$C: \left\{\begin{matrix}\dot{k} > 0 \\ \dot{p}_1 > 0\end{matrix}\right\} \quad [\frac{a}{b}]^{\frac{a}{1-a}} \frac{p_2}{b(a-1)} e^{-rt} < p_1(t) < [\frac{a}{ab}]^{\frac{a}{1-a}} \frac{p_2}{b(a-1)}\, e^{-rt} \tag{24}$$

The point (D) where $p_1(t) = 1$ and $\dot{p}_1 = 0$ can be regarded as a stable singular point of solution.

The shaded areas represent infeasible trajectories since movement is away from the terminal condition, $p_1 = 1$. Optimal trajectories where p_1 moves along the line $\dot{p}_1 = 0$ to $p_1(T) = 1$ are not allowed (except for point D) since p_2 is constant.

Furthermore, from Eq. (17) it can be shown, using Eqs. (18), that optimal investment trajectories always increase over time. This follows from:

$$\frac{d}{dt} \log u = a\, [\frac{(a-1)b}{p_2}]^{\frac{1}{a}} (\frac{1}{a} - 1) + \frac{r}{a} \tag{25}$$

The constant a is positive as is the factor $[\frac{(a-1)b}{p_2}]^{\frac{1}{a}}$, since $p_2 < 0$. As a

result of the fact that

$$0 < \alpha < 1$$

it follows that

$$\frac{\dot{u}}{u} > 0 \qquad (26)$$

We shall explore this general solution further for two simple examples, which differ only in the value assumed for the production elasticity.

Elasticity:	$\alpha = .5$ (example 1)
	$\alpha = .25$ (example 2)
Constant:	$b = 1$
Initial capital:	$k_0 = 100$
Time period:	$T = 10$
Discount rate	$r = 0$
Forgetting factor	$a = .25$

The equations of human capital and costate can be expressed analytically.

For an elasticity of $\alpha = .5$ these optimal trajectories are:

$$\ln k(t) = .25t + 2 \ln \{ 1 - (p_2 + 1)e^{.25(T-t)} \} + \ln k_0 - 2 \ln \{1 - (p_2 + 1) e^{.25T}\} \qquad (27)$$

and

$$p_1 = \left[\frac{1}{-\frac{1}{p_2} + \exp\{.25(T-t)\}\ (1 + \frac{1}{p_2})}\right] \qquad (28)$$

The costate trajectory is sketched in Figure 2 for different budget values. [Figure 2 here]. The three areas identified in Figure 1 are also delineated in Figure 2. The optimal investment trajectories for the given set of budget values are shown in Figure 3. [Figure 3 here]. Both Figures 2 and 3 are in agreement with the general findings, mentioned above, that

(1) the sign $(\dot{p}_1)$ does not change over the planning trajectory;

(2) sign $(\frac{\dot{u}}{u})$ is positive.

The second example, where the elasticity is .25 exhibits the same features. The optimal trajectories are described by the equations:

$$\ln k(t) = .75t + .75 \ln\{1 - [1 + (\tfrac{\dot{u}}{3} p_2)^3]\, e^{.75(T-t)}\} + \ln k_0 - .75\{1 - [1 + (\tfrac{\dot{u}}{3} p_2)^3]\, e^{.75T}\} \qquad (29)$$

and

$$p_1 = \left[\frac{1}{-(\frac{.75}{p_2})^3 + \{1 + (\frac{.75}{p_2})^3\}\, e^{.75(T-t)}}\right]^{1/3} \qquad (30)$$

Optimal costate and investment trajectories for this example are given in Figures 4 and 5 respectively. [Figures 4 and 5 here]

Both examples confirm the earlier statement that optimal investment trajectories call for increasing investments over time. In fact the numerical examples indicate trajectories which are exponentially increasing.

5 Conclusions and Summary

The problem studied in this paper is how schools can maximize the amount of human capital embodied in a student at the end of some specified schooling period, given a fixed budget. The human capital accumulation equation consists of two terms:

(i) a constant rate of obsolescence on forgetting applied to previously accumulated knowledge or human capital, and

(ii) a production function for new human capital.

The analytical solution to this problem supports the present practice of investing more in the later than the earlier years. The optimal investment trajectories are found to increase with time, although in the numerical examples the rate of change of the trajectory slope is smaller the smaller the budget and the higher the production elasticity.

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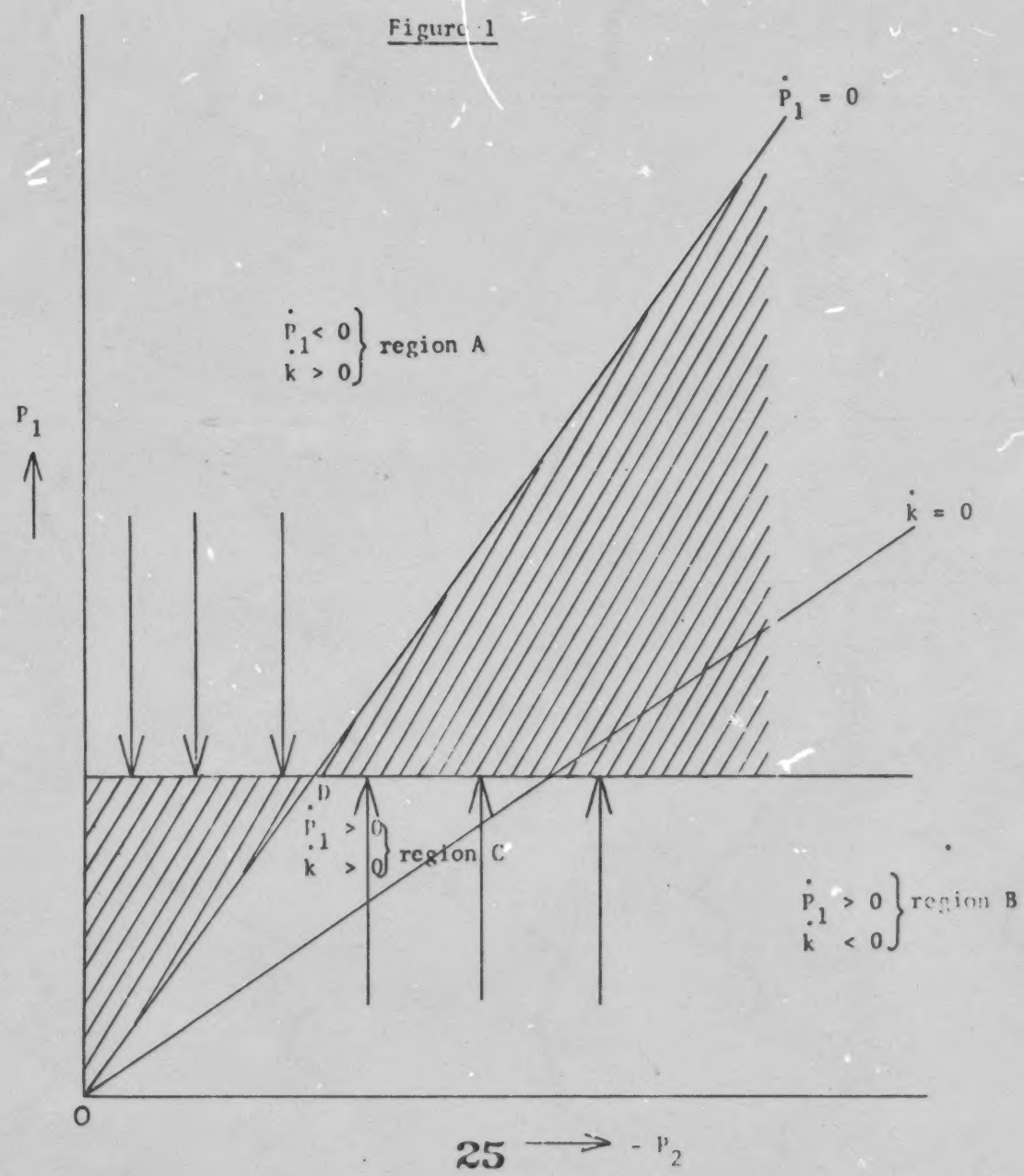
Abstract

Considers the maximization of human capital at the end of a given schooling period for an individual student, when a fixed budget is available. Human capital accumulates through a generation of new knowledge on the one hand and forgetting or obsolescence of old knowledge on the other. The new knowledge production function has been assumed to be of the Cobb Douglas type with the stock of human capital and new investments as "factors of production." The optimal investment trajectories are found to imply an increase in spending over the schooling period, whatever the size of the budget available.

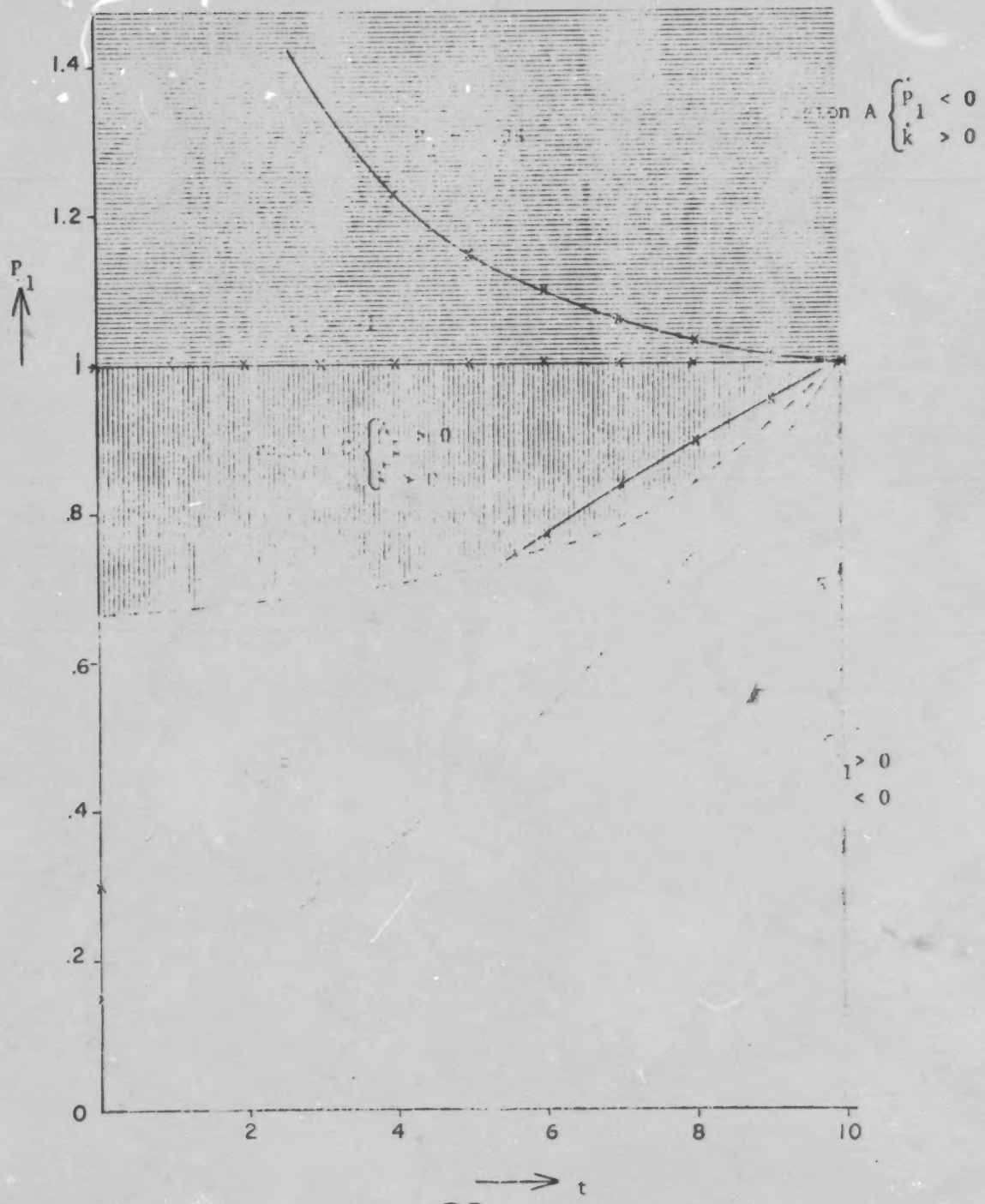
Directions of the Stock of Human Capital
and the Price of Net New Human Capital

Figure 1



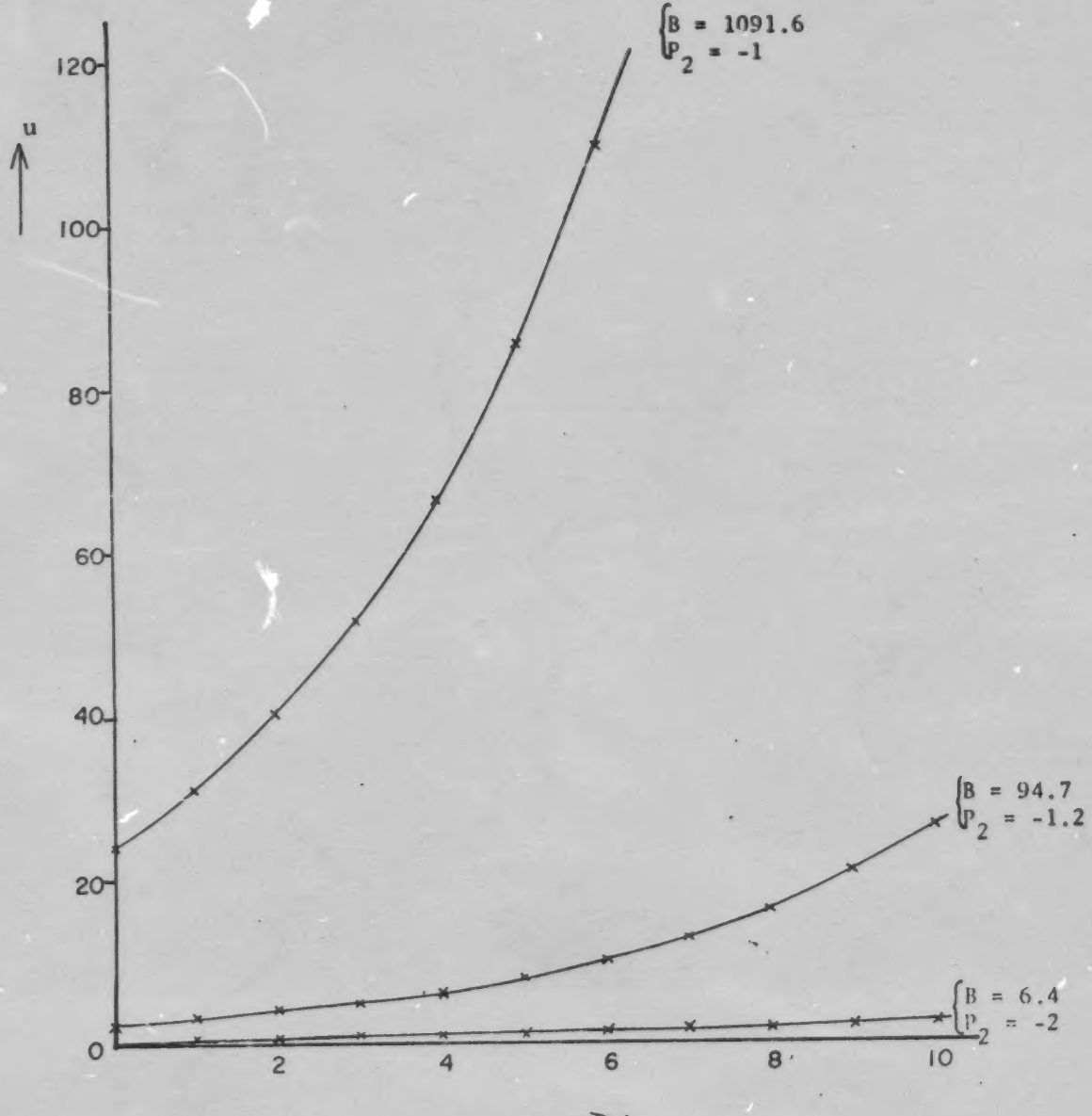
Optimal Trajectories of the Price of Net New Human Capital,
Elasticity .5, for Different Budget Values

Figure 2



Optimal Investment Trajectories, Elasticity .5,
for Different Budget Values

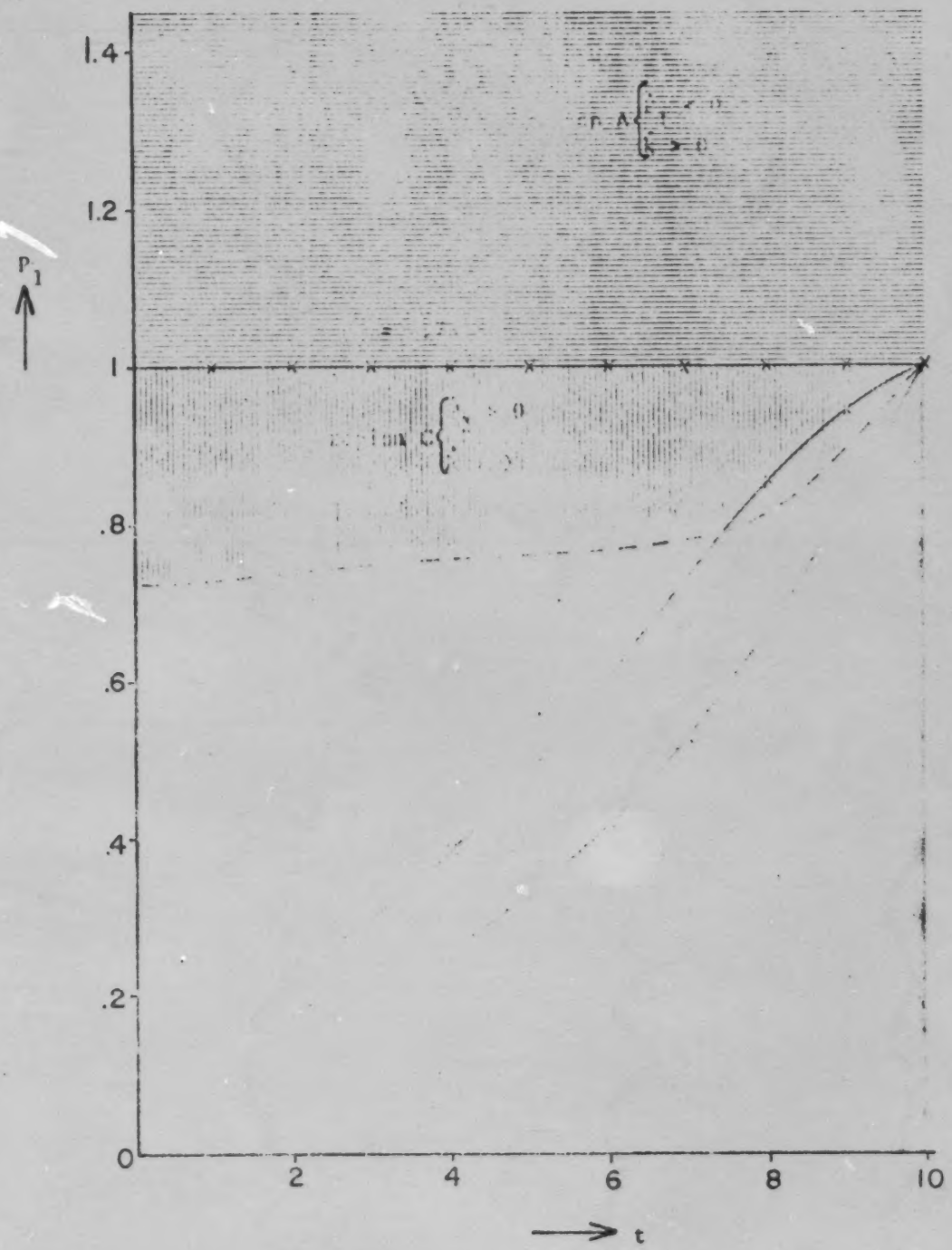
Figure 3



* The trajectory for $P_2 = -.95$ is beyond the scale of this graph.

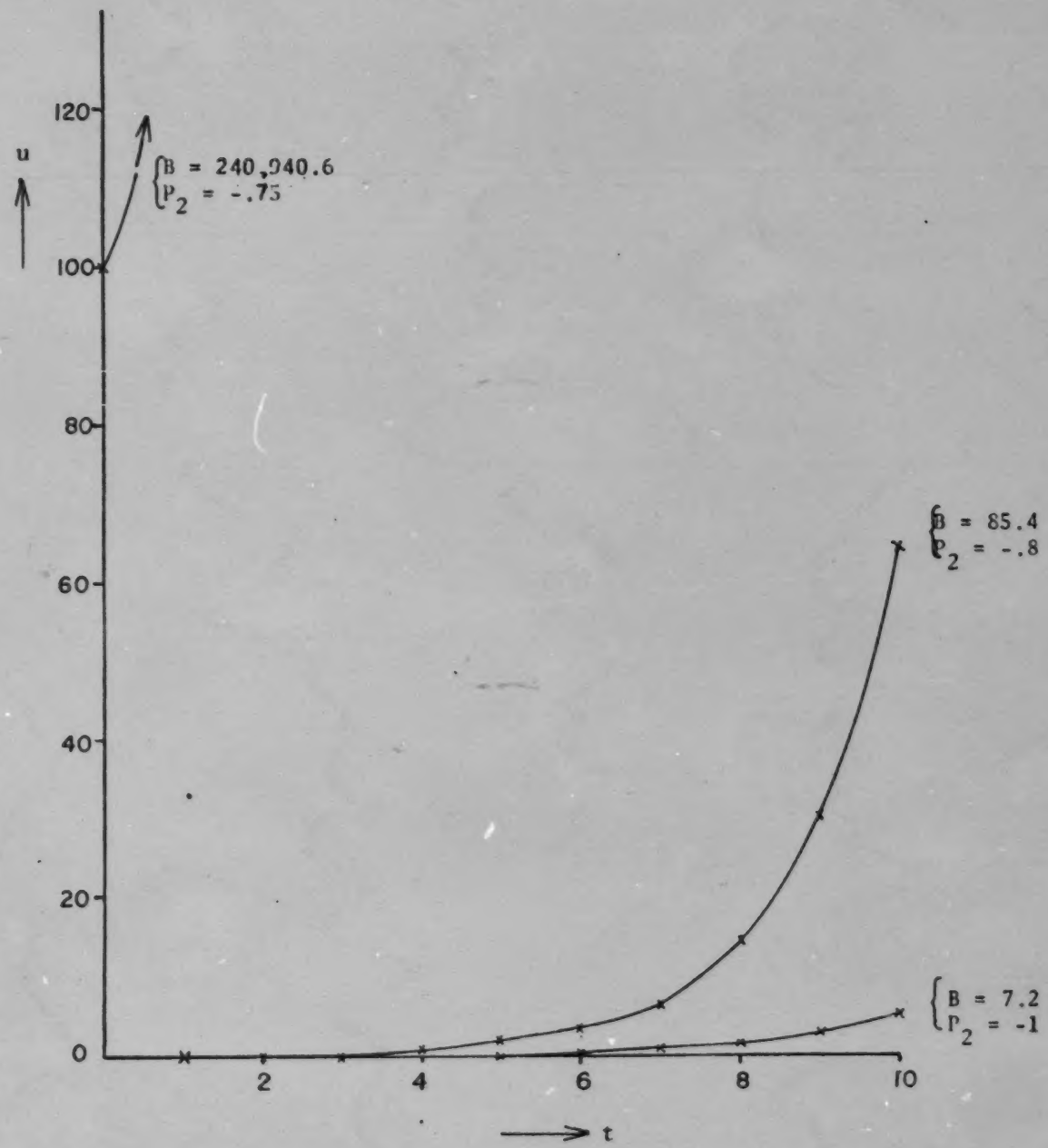
Optimal Trajectories of the Price of Net New Human Capital,
Elasticity .25, for Different Budget Values

Figure 4



Optimal Investment Trajectories, Elasticity .25,
for Different Budget Values

Figure 5



APPENDIX B

THE PRODUCTION OF HUMAN CAPITAL OVER TIME*

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(Revised Version)

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ABSTRACT

The arguments for early childhood intervention programs in education implicitly assume (i) the elasticity of final human capital stock with respect to initial human capital stock is greater than one and/or (ii) the elasticity of final capital stock with respect to purchased school inputs decreases over the school-life. This paper develops a model of human capital accumulation in the schools which enables us to estimate these elasticities. Using longitudinal data on cohorts of pupils to estimate the model, we find some support for early childhood programs for disadvantaged children but no support for such programs for advantaged children. For the latter, the optimal pattern of resource allocation over time is one where the level of purchased school inputs increases with time.

A decade has passed since the passage of the federal Elementary and Secondary Education Act [ESEA] of 1965. Current federal expenditures for compensatory education under that Act now exceed one and a half billion dollars annually. Those expenditures are largely focused on children in the elementary grades, in particular, kindergarten through grade three.[1]

A major objective of compensatory education programs can be viewed as reducing poverty by increasing the human capital stocks of students at the end of their school-lives (i.e., the terminal human capital stock). Proponents of early childhood intervention programs expect terminal human capital stocks to be higher when limited resources are reallocated from later to earlier grade levels. However, empirical research has by and large failed to fulfill this expectation. Evaluations of early childhood intervention programs like Headstart have generally produced inconclusive results [Cicirelli (1969), Bronfenbrenner (1974), Ryan (1974)].[2]

There are several possible explanations for this finding, but the one explored in this research is that the elasticity of terminal capital with respect to school inputs received in the early grades is very low. While the conventional belief among psychologists and educators has been that this elasticity is high [Hunt (1961), Bloom (1964)], others have argued that the elasticity of terminal capital with respect to school inputs is higher in the adolescent years than the early childhood years [Rohwer (1971)]. If the latter view is correct, compensatory education programs should maximize terminal capital by increasing school resources in the secondary grades, not the early elementary grades.

The purpose of this paper is to estimate the productivity of school inputs over time for both "advantaged" and "disadvantaged" children. The estimates obtained will have implications for the optimal allocation of limited school resources over the school-life.[3] In the case of disadvantaged children, the estimates provide one possible explanation for the presumed failure of compensatory, early childhood education programs. In the case of advantaged children, the results offer one prediction of the success of non-compensatory, early childhood education, which is receiving increasing political support these days.

The model of human capital accumulation is derived and described in the next section, followed by a discussion of the data and sample used in the estimation of the model. Subsequently, the estimated results are presented, and the policy implications of the results are explored.

I. THE MODEL OF HUMAN CAPITAL ACCUMULATION

The voting citizen is the ultimate decision-maker in public education. One of the objectives which could be attributed to voters, and thus the schools, is maximization of individual stocks of human capital upon termination of the formal schooling period.[4] That is the objective assumed in this paper, although voters might also have preferences with respect to the level of human capital stocks over the school-life or with respect to the variance between pupils in the size of the terminal human capital stock.

Given the voters' objective, the optimal path of resource allocation over time depends on the per pupil budget over the school-life and the production function for human capital. The size of the budget is assumed exogenous in this paper. We focus on the production function of human capital over time.

The production of the terminal human capital stock is perhaps most appropriately represented by a "goods in process" model [Haavelmo (1960)].[5] New learning by students in a given period of time is a function of their

existing human capital stocks and the current flow of inputs. Hence, the production function for the terminal capital stock can be expressed as a function of the stock upon entering school (the initial capital stock) and the levels of inputs over the school-life.

Learning theory provides little guidance for specification of the production function in education, either in terms of the variables which should be included or the functional form which should be adopted. Some guidance for specification of a human capital production function is provided by earlier theoretical and empirical studies. The model adopted here most resembles the one presented by Ben-Porath (1967).

Ben-Porath assumes the production function for human capital to be Cobb-Douglas with decreasing returns to scale such that

$$Q_t = A\,(S_t\,K_t)^{\alpha}\,X_t^{\gamma} \qquad (1)$$

where α, $\gamma > 0$, Q is the flow of output, X is the quantity of purchased inputs, and s is the proportion of the stock of human capital used in the production of new human capital.

Our specification of the production function differs from Ben-Porath's in several ways. No constraint is imposed on returns to scale, s is assumed to be constant over time, and γ is assumed to vary over time. Furthermore, limitations of our data force us to use a different measure of output flow. Whereas Ben-Porath's measure is in terms of physical units of new capital produced (which is equivalent to the final level of capital stock minus the depreciated stock of initial capital), the measure used here is the *ratio* of the final level of capital stock to the depreciated stock of initial capital. This measure of output enables us to express terminal capital (K_t) in terms only of the capital stock upon entering school (K_0) and input levels over time.

The production function used here also disaggregates purchased inputs into home inputs, Z, and school inputs, X:

$$\frac{K_t}{(1 - \delta) K_{t-1}} = A K_{t-1}^{\alpha} Z_t^{\beta} X_t^{\gamma_t} \qquad (2)$$

where $X_t > 0$. There are no observations in our sample where $X_t \leq 0$, i.e. where the student did not attend school in any one period. Eq. (2) can be rewritten:

$$K_t = (1 - \delta) A K_{t-1}^{\alpha+1} Z_t^{\beta} X_t^{\gamma_t} \qquad (3)$$

Hence, $\alpha + 1$, β, and γ_t are interpreted as elasticities of the final capital stock, K_t, with respect to the initial capital stock, K_{t-1}, home inputs, and school inputs respectively. The initial human capital stock in each period represents the human capital available to the child to combine with other inputs in producing the final capital stock. The coefficient A represents the state of technology, and δ represents the rate of depreciation, but since both are constants, neither can be identified.[6]

Since data on purchased school inputs is available over the school-life of each pupil in the sample, a time path of the corresponding production elasticities, γ_t, can be estimated. Unfortunately, we lack data on how the capital stock and home inputs vary over the school-life of the child. Hence the exponents, $\alpha + 1$ and β, on those variables are constrained to be constant over time in this model. This constraint may bias the estimated time-pattern of elasticities on school inputs if in the true model α and β are time dependent. However, since we have no *a priori* information on how those elasticities change over time, we cannot determine the direction or size of the possible bias.

Employing Eq. (3) to express K_T in K_0 and the sequence of school inputs $X_0, X_1, \ldots, X_{T-1}$ we derive the equation to be estimated in this paper:

$$K_T = A' K_0^{\alpha'} X^{\beta'} \prod_{i=0}^{T-1} X_i^{\gamma_i'} \tag{4}$$

where:

$$A' = [(1 - \delta) A]^{\left(\sum_{i=0}^{T-1} (\alpha + 1)^i\right)}$$

$$\alpha' = (\alpha + 1)^T \tag{5}$$

$$\beta' = \beta \sum_{i=0}^{T-1} (\alpha + 1)^i$$

and

$$\gamma_i' = (\alpha + 1)^{(T-i-1)} \gamma_i .$$

Hypotheses Related to Early Childhood Education

The hypotheses implicit in the arguments for early childhood intervention programs can be tested in the context of our model as represented by Eq. (3). Thy hypotheses can be best developed by considering an example. The impact of changes in purchased school inputs (X_1) at grade one on the final capital stock at, say, grade two depends on (i) the elasticity (γ_1) of capital stock (K_1) at grade one with respect to purchased inputs (X_1) at grade one and (ii) the elasticity ($\alpha + 1$) of final capital stock (K_2) at grade two with respect to the capital stock at grade one (K_1). Using Eq. (3) we can quickly compute the elasticity of capital stock at grade two with respect to purchased inputs at grade one as $\gamma_1(\alpha + 1)$.

If the exponent $(\alpha + 1)$ is greater than one, *ceteris paribus*, an increase in purchased inputs at grade one results in a higher percentage change in the final capital stock at grade two than grade one. Consequently, one possible economic interpretation of the arguments for early childhood intervention programs is $\alpha > 0$, which implies increasing marginal productivity of the initial stock of human capital. The null hypothesis which we test later in this paper is $\alpha = 0$.

A second possible interpretation of the arguments for early childhood programs is that the exponent γ_t decreases over the school-life of the child. If γ_t is a linear function of time such that $\gamma_t = c_0 + c_1 t$ the policy implication is, *ceteris paribus*, the smaller is c_1 the more resources should be allocated to the early grades. Hence, we test the null hypothesis of a constant elasticity on purchased school inputs over time, i.e., $\gamma_t = c_0$ for all t.

II. SPECIFICATION OF THE MODEL

A. The Sample and Data

The sample used in estimation of the human capital accumulation model as represented by Eq. (4) consists of 669 students who completed eighth grade by 1965 and attended schools in a single district in California for all eight years. Of the 669 students, 356 are black and 313 are white, 212 are in the college preparatory track in junior high school and 457 are in the business-vocational track.

Race may be considered a proxy for income in this sample. According to 1960 Census figures, a year when students in the sample were in elementary school, mean income of black families was $5287 and mean income of white families was $7768 in the geographic area included in the school district under study. No measures of income were available for the sample used, but in terms of possible surrogates we find an index of home inputs is 5.17 for white pupils

and 4.25 for black pupils, and the number of siblings living at home is 1.38 for whites and 2.85 for blacks. The differences are statistically significant.[7]

Track in school may be considered a proxy for achievement or ability in this sample. Upon entering junior high school, students are assigned to tracks on the basis of course grades and teacher recommendations. The mean percentile I.Q. score of pupils in the college track is higher than the mean score of pupils in the business-vocational track at both grades one and eight, and the difference between those scores increases from eight points at grade one to nineteen points at grade eight. A disproportionate number of students in the college preparatory track are white and a disproportionate number of students in the business-vocational track are black.

The data collected on the sample includes measures of the stock of human capital at grades one and eight, measures of home inputs collected at grade eight, and measures of purchased school inputs between grades one and eight. The precise specification of these variables is provided in Table I. Of all the variables included in the specification of the model, the one presenting the greatest conceptual difficulties is the measure of human capital stock.

B. Measuring the Human Capital Stock

Human capital is measured in this paper by percentile scores on standardized examinations of cognitive learning. Implicit in this measurement of human capital are the assumptions that cognitive knowledge can be measured by scores on standardized examinations and that cognitive knowledge is an important determinant of the present value of one's future earning stream. The validity of these assumptions is discussed later.

The specific measures of human capital stock used in this paper include the percentile I.Q. score at grade one and percentile scores on I.Q. and verbal

achievement tests at grade eight. To test the robustness of the estimated structure of the model, three measures of grade eight capital stock are utilized: Hemnon-Nelson I.Q. (Q_1), Stanford Language (Q_2), and the Differential Aptitude Test of Reading (Q_3).

C. Home Inputs

The quantity of instructional services provided in the home is assumed here to be a function of the capital and labor in the home. By capital we mean both physical capital such as books, games, toys, etc. and human capital of the parents. By labor we mean the quantity of time parents spend with children as well as the amount of time children spend studying on their own.

Physical capital is proxied in this study by (i) an index, compiled from student questionaire responses, of the number of cultural items in the home and (ii) a variable indicating home ownership. The index ranges in value from zero to seven. The home ownership variable is dichotomous, taking the value e, the base of the Naperian logarithm, if the family owns its home and one otherwise.

Human capital is proxied by the number of years of education of the mother and the father, as reported by the student. We have no measure of the time input of children, but the time input of parents is proxied by the number of siblings in the home.[8] The reasoning here is that, *ceteris paribus*, the larger the number of children in the home, the smaller is the amount of time the parents can spend with any one child. The time of older siblings may in this respect be a substitute for parental time, but we have no data on the age-ordering of siblings.

D. School Inputs

The quantity of instructional services provided in the school can be assumed to be a function of the capital and labor in the school. Capital includes physical capital such as books, laboratory equipment, special educational aids, etc. and

human capital of the teachers, administrators, and other personnel. At a micro level, labor in the school refers to the quantity of time a teacher allocates to a given child and the pupil's own work effort. Teacher time may be related to class size.

While it would be desirable to have direct measures of capital and labor in the school, we proxy these purchased inputs by current real expenditures per pupil.[9] This aggregate measure of school inputs is also a convenient one to use in the estimation of our model. Including the individual components of expenditures in this model might result in multicollinearity if the components are highly correlated over time.

The expenditure variable was computed from (i) school budget records, which gave expenditures on supplies and equipment, administrative and counseling personnel salaries, (ii) teacher personnel files, which provided information on salaries of specific teachers, (iii) school attendance records, which reported class sizes for each teacher, and (iv) student academic records, which included information on specific schools and classes of pupils. For the particular sample used in this study, expenditures per pupil are relatively constant over the elementary years but increase sharply when students transfer to junior high school.[10]

No direct measure of the pupil's own work effort is available to us, but a determinant of that work effort is available. Pupils can be viewed as making work-leisure choices within the classroom. If the returns to work, changes in future income and current rewards from parents and peers, increase, the child could be expected to increase his work effort. If the socio-economic composition of the peer group changes, current rewards change, and the child may change the proportion of time spent in study as opposed to leisure. Hence, we include average proportion of low school peers of low economic status in grades one through eight as a non-purchased input which could be expected to affect the

child's work effort and ultimately his terminal capital stock.[11] An increase in the proportion of low economic status peers may also adversely affect learning by requiring the teacher to spend more time in discipline and less in instruction.

III. RESULTS

The estimated parameters of the non-linear capital accumulation model given in Eq. (4) are reported in Table I. The structure was estimated using ordinary least squares. The time paths of elasticities associated with purchased inputs were constrained to fit a first-degree polynomial, which restricts the γ_i' exponents to a straight line.[12] The time path of γ_i exponents is similarly restricted only if $\alpha = 0.0$. Two reasons lie behind this specification of the time path. First, we are attempting to answer a relatively simple policy question: should schools allocate equal quantities of school inputs to each grade level? The corresponding null hypothesis is that production elasticities are constant over time with the alternative hypothesis being they are not constant. Hence, we are primarily interested in the *direction* of change of production parameters over time. Second, preliminary results indicated that alternative specifications of the time pattern fit the data no better; indeed, coefficients on higher degree polynomial terms in the distributed lag specification were often statistically insignificant.[13]

A. Pre-School

The estimated exponent on the pre-school level of human capital stock (K_0) is .631. In other words, a one unit increase in the pre-school stock is reflected in only a .53 unit increase (computed at the means of K_0 and K_T) in the terminal stock. The corresponding value of α computed from Eq. (5) is -.064, which is less than zero and implies diminishing marginal productivity.

B. Home Inputs

The exponents on home input variables have the expected signs. The proxies for physical capital in the home, the index of cultural items and family ownership of the home, are both positively related to changes in the human capital stock. At the means of the variables, a one unit change in the cultural index is reflected in a .16 unit change in the terminal human capital stock. Ownership of the home is associated with a terminal capital stock which is 2.28 points higher.

The proxy for human capital in the home, parental education, is positively related to changes in the pupil's capital stock for both parents, but the exponent is statistically significant only for education of the father. An increase of the father's education by one year results in a .75 unit change in the pupil's terminal capital stock. If we had any *a priori* expectation that these exponents should differ between parents, it would be that the exponent should be larger for the mother, since she probably spends more time with the children than does the father. This is the result obtained by Leibowitz (1974) in her study of preschool investment in children. We cannot reject the null hypothesis in this study that the exponent is of equal size for mother and father.

The proxy for labor in the home, number of siblings, is as expected, negatively related to changes in the pupil's capital stock. An increase in the number of siblings by one is reflected in a decrease in terminal capital of .56 percentile points. This result is similar to one found by Bowles (1970).

C. School Inputs

School variables include purchased and non-purchased inputs. The latter variable is the proportion of peers of low economic status in grades 1-8 which is negatively related to changes in the capital stock. However, the results indicate that decreasing the variable by 10% would have a cumulative impact on the

terminal capital stock of only + 0.6%.

The exponents on the purchased input variables in Table I imply a time path of production elasticities (γ), which is reported in the first column of Table II. The pattern is not constrained to always be above zero; hence, negative estimated elasticities are possible. However, the negative exponents estimated are unrealistic and probably result from the restrictions imposed on the pattern of exponents over time; they should be regarded as simply being close to zero.

The elasticity increases from .018 in grade one to .198 in grade eight. A given percentage increase in purchased inputs is estimated to produce a larger percentage increase in the capital stock at grade eight than at grade one. Since $\alpha < 0$, the conclusion follows that a given percentage increase in purchased inputs at grade one results in a smaller percentage change in terminal capital (K_T) than an identical percentage increase in purchased inputs at grade eight.

The time pattern of elasticities (γ) estimated for Q_1 is displayed graphically in Figure 1. We test the robustness of these results by using two alternative measures of capital stock, the Stanford Language Test (Q_2) and the Differential Aptitude Test of Reading (Q_3). Only the parameters α and γ are reported here because the exponents estimated for other variables in the model changed vary little in terms of sign and statistical significance.[14] The results are given in Table 2 and drawn in Figure 1.

For capital measures Q_2 and Q_3 we find an estimate of α which is slightly greater than zero. All the estimated time patterns of γ have the same direction of change as for Q_1, although the rate of change varies somehwat. Taking these results together [Eq. (5)], we find the elasticity (γ') of terminal capital stock with respect to purchased inputs (X_t) increases with the grade level. The results

obtained using Q_1 are relatively robust; hence, all further estimation was done using Q_1 as the measure of the terminal capital stock.

D. Disaggregation by Race and Track

The sample was disaggregated by race, and Eq. (4) was re-estimated separately for blacks and whites to determine whether or not the time pattern of elasticities on purchased school inputs differ. As noted earlier, race is largely a proxy for income for this sample. The results are again reported in Table 2 and displayed in Figure 2. The estimates of α are slightly less than zero for both blacks and whites.

The estimated time pattern of elasticities (γ and γ') on purchased school inputs increase over the school-life for whites. For the black sample, we cannot reject the null hypothesis that the value of γ' is constant between grades 1 and 8.

The sample was also disaggregated by track in school and the model re-estimated separately for pupils in the college preparatory track and pupils in the business-vocational track. Pupils in the college track are relatively high achievers, while pupils in the business-vocational track are relatively low achievers. Again, the estimates of α are less than zero for both sub-samples. The estimated elasticities (γ and γ') increase with time for both sub-samples, although the results are statistically significant for the college track only. If the elasticity on purchased school inputs is constrained to be constant over time, the estimates become statistically significant for the business-vocational sub-sample. These results are again shown in Table 2 and Figure 2.[15]

The differences in time patterns of elasticities between tracks may be a result of different learning objectives as well as different student abilities. The assumed objective in this study is verbal cognitive achievement, but it is possible that the school may not be so interested in imparting verbal skills to

vocational track students as it is interested in imparting machine-working skills.

Summing up the results, we find the estimates of α are always close to zero, irrespective of output measure or sub-sample employed. The production elasticities (γ) associated with purchased inputs increase over the eight years of schooling for the full sample. The slopes, however, differ substantially for different measures of the human capital stock and for different sub-samples. In general, we find the slope is higher for a measure of reading achievement (Q_3) than a measure of I.Q. (Q_1), higher for whites (high income) than blacks (low income), and higher for students in the college preparatory track (high achievers) than students in the business-vocational track (low achievers).

IV. POLICY IMPLICATIONS

A. A *Caveat*

Prior to discussing the policy implications of the empirical results, a *caveat* is in order. The results obtained here may not be generalizeable to the world as a whole. These results may be peculiar to the sample used, the measures of human capital stock used, and the types of inputs typically purchased and teaching technology typically employed in the district studied.

The sample is probably not atypical of students enrolled in large, urban school systems, but probably has little in common with school children in rural Iowa. Disaggregation by race and track in school indicates the results can and do differ between sub-samples.

Measurement of the human capital stock is a more serious problem. Implicit in the measure of human capital stock employed in this paper are two assumptions. First, cognitive knowledge is assumed to be measured by scores on standardized examinations, including I.Q. tests. A difficulty with such standardized exams is that they reflect only the level of learning relative to

some comparison group (in our case a national sample of pupils) instead of absolute level of knowledge.[16] Furthermore, the precise skills and knowledge measured changes over the school life of a child. Measuring gains in cognitive knowledge, or educational value-added, by comparing percentile scores at, say, grades one and eight assumes the same type of knowledge is being measured, which is not precisely true.

Second, cognitive knowledge is assumed to be an important determinant of the present value of one's future earning stream. If both these assumptions hold true, one would expect to find a statistically significant, independent relationship between scores on tests of cognitive learning and future income. Recent research [Griliches and Mason (1972), Hause (1972)] has found such a relationship to exist, although its size is small. The size of the relationship is larger if one explicitly takes into account the effects of test scores in determining years of education attained by individuals [Ribich and Murphy (1973)].

A further problem with the examinations used in this study is that they are all verbal in emphasis. Thus, the results obtained here do not necessarily extend to other types of cognitive knowledge. For example, learning mathematics is more structured than learning verbal skills, requiring a firm grasp of concepts at one level before the student can proceed to the next level. One might find a different time pattern of production coefficients for mathematics achievement than was found for verbal achievement.

In addition to the school output, home and school inputs may be imperfectly measured. Direct observations and measurements of capital and labor inputs in the home are ideally required for our empirical analysis. At the very least, it would be desirable to have information from parents on the amount of time they spend with children (for example, see Leibowitz (1974)) and the types of physical objects available in the home. Unfortunately, we have to rely upon student responses to questionnaire items, which serve as proxies for capital and labor

inputs.

Direct measures of capital and labor inputs in the school would also be desirable. While proxies for these inputs are available, we have little information on the reliability of those proxies.

Lastly, the model says nothing about the effects of changing technology and increasing expenditures in the early grades. If early childhood intervention programs entail radically different teaching technologies, the parameters estimated in this paper are likely to provide inaccurate predictions of the results of such programs. Furthermore, the patterns of production elasticities estimated here may in part reflect differences in teaching technologies between elementary and junior high schools.

B. Hypothesis Testing

We earlier formulated two hypotheses implicit in the arguments for early childhood intervention programs. The first null hypothesis tested is that the elasticity $(\alpha + 1)$ of the final capital stock in one period with respect to the capital stock in the preceding period is equal to one. In other words, the null hypothesis is $\alpha = 0$. We find we cannot reject that null hypothesis for the full sample, any of the sub-samples, or any of the measures of terminal capital stock. This finding provides no support for the proponents of early childhood programs.

The second null hypothesis is that the elasticity (γ) of the final capital stock in one period with respect to purchased school inputs in that period is of equal value for all periods. In other words, the null hypothesis is γ_t is constant for all t. The null hypothesis is rejected for the full sample and for what we assume are high achieving and high income students. We accept the alternative hypothesis that value of γ increases with time. Hence, this finding provides no support for those advocating special early childhood programs for

all children.

However, this second null hypothesis cannot be rejected for what we assume are low achieving and low income students. These are precisely the children who are the targets of early childhood compensatory education programs. Hence, our findings provide some support for such programs, although it should be kept in mind that we never found the production elasticities γ to decline with time. This latter result would have provided the strongest support for early childhood programs. Furthermore the values obtained for γ in the early grades are small ($\gamma_1 = .13$ for blacks and $\gamma_1 = .01$ for business-vocational track) which indicates increasing purchased school inputs has a relatively small effect on achievement.

C. Optimal Resource Allocation Over Time

The hypothesis testing provides some general conclusions about optimal resource allocation over time. The precise implications of our findings for resource allocation can be derived by assuming the school attempts to maximize terminal human capital subject to a budget constraint. The optimal investment trajectory, assuming a zero rate of discount, is obtained by setting up the Lagrangian:[17]

$$L = A' K_0^{\alpha'} Z^{\beta'} \prod_{i=0}^{T-1} X_i^{\gamma_i'} + \lambda \left(B - \sum_{i=0}^{T-1} X_i \right) \qquad (6)$$

The first-order conditions for the problem are then $\frac{\partial L}{\partial X_i} = 0$ such that

$$\frac{\gamma_i'}{X_i} = \frac{\lambda}{K_T}$$

for all i , so that (7)

$$\frac{\gamma_1'}{X_1} = \frac{\gamma_2'}{X_2} = \ldots = \frac{\gamma_T'}{X_T}$$

In other words, the pattern of investments should be in the same direction as the pattern of change in the elasticities, or in a continuous time notation:

$$\text{sign } (\dot{X}) = \text{sign } (\dot{\gamma}_1') \qquad (8)$$

The same result is found by applying Pontryagin's maximum principle; the derivation is reported in the appendix to this paper.

In those cases where the statistically significant results obtained in the estimation of the model showed production elasticities which continuously increase with time between grades one and eight, the optimal investment trajectory should be one where the quantity of purchased inputs per pupil also increases with time. Comparing two time periods, i and j, Eq. (7) can be rewritten:

$$\frac{X_i}{X_j} = \frac{\gamma_i'}{\gamma_j'} = \frac{(\alpha+1)^{(T-i-1)}\,\gamma_i}{(\alpha+1)^{(T-j-1)}\,\gamma_j} = (\alpha+1)^{(j-i)} \cdot \frac{\gamma_i}{\gamma_j} . \qquad (9)$$

For the case where output is Q_1 and the full sample is used to estimate the structure, the optimal investment trajectory is one where about seven times as many purchased inputs are given to students in grade eight as are given to students in grade one. This ratio considerably exceeds the actual one as demonstrated in Figure 1. Marginal productivities computed at the means of the variables for the same sample and output indicate that increasing purchased inputs by \$100 in grade eight or \$100 in grade one would increase terminal achievement (K_T) by 2.5 and .3 units respectively. In other words, one dollar spent in grade eight has the same marginal effect on K_T as does about \$8.33 spent in grade one.

While the conclusion that school inputs should increase with time appears to be generally true for whites and high achieving students who are in the college preparatory track, it does not hold for blacks or students in the business and vocational tracks. For these latter sub-samples resources should be more nearly equally distributed over grade levels, which implies the actual expenditure pattern in the school district should be flattened out with an increase in dollars spent in the early grades and a decrease in dollars spent in the later grades.

In sum, this paper provides some support for early childhood intervention programs oriented towards disadvantaged children, but it does not provide support for extending early childhood programs to all children. In the specific district studied, the terminal human capital stock of disadvantaged children could have been increased by reallocating resources from the later grades to the earlier grades. The opposite conclusion holds for advantaged children.

APPENDIX

In continuous time the problem of maximizing terminal time human capital stock subject to a budget constraint can be expressed as maximization of

$$\int_0^T \dot{K}(t)dt \qquad \text{(A-1)}$$

where

$$\dot{K} = AK_t^{\alpha} X_t^{\gamma t} \qquad \text{(A-2)}$$

and subject to the budget constraint:

$$B = \int_0^T X_t \, dt \quad . \qquad \text{(A-3)}$$

The budget constraint can be transformed into a differential equation:

$$\dot{R} = - X \qquad \text{(A-4)}$$

with $R(0) = B$ and $R(T) = 0$.

For the time being we ignore non-negativity constraints on the stock of human capital and on investments as well as constraints on the amount of positive investment. The Hamiltonian is then:

$$H = p_1 AK^{\alpha} X^{\gamma t} - p_2 X \quad . \qquad \text{(A-5)}$$

The costates are defined by the differential equations:

$$\frac{\dot{p}_1}{p_1} = - \alpha AK^{\alpha-1} X^{\gamma t} \qquad \text{(A-6)}$$

$$\dot{p}_2 = 0 \quad . \qquad \text{(A-7)}$$

The transversality condition resulting from the objective of maximizing terminal time human capital stock is

$$p_1(T) = 1 \quad . \tag{A-8}$$

The optimal investments are then determined by:

$$\frac{\partial H}{\partial X} = 0 \qquad \text{or} \qquad X = \frac{p_1 \gamma_t A K^{\alpha}}{p_2}^{\frac{1}{1-\gamma_t}} \quad . \tag{A-9}$$

From Eq. (A-9) it now follows for $\gamma_t > 0$ that

$$\text{sign } (p_1) = \text{sign } (p_2) \quad . \tag{A-10}$$

Since p_2 is constant, this implies that p_1 does not change sign. Combining Eq. (A-10) with the transversality condition of Eq. (A-8) and excluding the "infinite" budget case where $p_2 = 0$, we conclude that

$$p_{1,t} > 0 \ , \qquad p_2 > 0 \quad . \tag{A-11}$$

We can now proceed to explore possible upward or downward tendencies in the optimal investment schedule. Differentiate Eq. (A-9) with respect to time:

$$\dot{X} = \frac{1}{1-\gamma_t}\left[\frac{A}{p_2}\right]^{\frac{1}{1-\gamma_t}}\left[p_{1,t}\,\gamma_t K^{\alpha}\right]^{\frac{\gamma_t}{1-\gamma_t}}\left[\dot{p}_1 \gamma_t K^{\alpha} + \alpha p_1 \gamma_t K^{\alpha-1}\dot{K}\right]$$

$$+ \frac{\partial X}{\partial \gamma}\dot{\gamma} \tag{A-12}$$

where

$$\dot{X} = \frac{\partial X}{\partial \gamma}\dot{\gamma} \quad . \tag{A-13}$$

It can easily be shown that $\frac{\partial X}{\partial \gamma} > 0$. Hence, it follows that

$$\text{sign } (\dot{X}) = (\dot{\gamma})$$

if

$$K \neq 0 \ , \quad p_1 \neq 0 \quad .$$

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FOOTNOTES

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[1]The source of these data is a Stanford Research Institute report titled, "Alternative Strategies for Compensatory Education." According to the California State Department of Education, well over half of all students enrolled in ESEA, Title I, in the state are in kindergarten through grade three.

[2]While evaluations of early childhood intervention programs typically find large short-run gains in learning, the long-run gains are statistically insignificant.

[3]There is a large body of research on production of human capital in specific grade levels of public schools [Hanushek (1971), Katzman (1968), Murnane (1974)] , but the specifications of the production functions and the measurements of human capital have differed sufficiently to make the results non-comparable. Hence, these studies have no policy implications for resource allocation over time in the schools.

[4]Of course, the rationale for government intervention in education is the existence of externalities, which are largely unmeasureable. It could, however, be argued that external as well as private pecuniary and non-pecuniary benefits are a positive function of the level of the human capital stock.

[5]Although there has been considerable theoretical research on optimal investment in human capital by individuals over the life-cycle [Ben-Porath (1967), von Weizsäcker (1967), McCabe (1975), Wallace and Ihnen (1975)], that research has only limited applicability in this paper. In the problem stated here, the government, not the individual, is the actor, the period of time during which investment can take place is clearly defined and constrained,

and the objective is maximization of the human capital stock, not individual utility.

[6]The data do not enable us to distinguish between the model presented here where depreciation is multiplicative ($(1-\delta) K_{t-1}$) and one where depreciation is exponential ($K_{t-1}^{1-\delta}$). In the latter model the corresponding coefficient on old capital in Eq. (4) is $1 - \delta + \alpha$, and the resulting estimate of that coefficient, instead of telling us the value of α merely indicates whether or not δ exceeds α. In other words, the results would indicate whether or not the rate of depreciation exceeds the contribution of initial capital to growth in the capital stock. The policy implications of our results are similar for both models.

[7]It should also be noted that while blacks and whites attended largely integrated junior high schools, the elementary schools they attended were largely segregated on the basis of race.

[8]See Bowles (1970) for the rationale behind the interpretation of some of these variables.

[9]The usual simultaneous equations problem in estimation of production functions appears not to be present for the cohort of students under study. It is sometimes argued that "good" teachers are assigned or assign themselves to teach the "best" students, but our estimates of a simultaneous equation model of student achievement and teacher allocation do not support this assertion for the sample under study. A recent study by Greenberg and McCall (1974) reaches the opposite conclusion for the city of San Diego. It should also be noted that the estimated production elasticities on purchased inputs may not be those of the most efficient schools but rather some average of efficient

and inefficient schools. This problem is explored in greater detain by Bowles (1970) and Levin (1974).

[10]School district decisions about allocation over grade levels are influenced by state grand-in-aid programs in education. For example, in California the amount of state aid per pupil is higher in secondary than elementary education. Furthermore, the sharp rise in expenditures at grades seven and eight is in part due to special property tax over-rides legislated by the state for the purpose of inducing districts to provide specialized education (where instructors teach their specialty as opposed to one instructor teaching all subjects). The pattern of expenditures for this sample is, also, in part explained by the fact that early childhood compensatory education programs were almost non-existent during the years the sample was in elementary school. All pupils in the sample had completed elementary school by 1963.

[11]Information on family income is not available for the sample of students used in this study. Data on educational and occupational status of parents was used to construct the variable measuring the proportion of school peers of low social status.

[12]Constraining the time pattern of the γ_i' coefficients to fit a first-degree polynomial, Eq. (4) can be rewritten:

$$K_T = A' K_0^{\alpha'} Z^{\beta'} \left[\pi X_i\right]^{\theta_0} \left[\pi X_i (T-i)\right]^{\theta_1}$$

Taking logs we obtain:

$$\log K_T = \log A' + \alpha' \log K_0 + \beta' \log Z + \theta_0 \sum_{i=0}^{T-1} \log X_i + \theta_1 \sum_{i=0}^{T-1} (T-i) \log X_i$$

where:

$$\gamma_i' = \theta_0 + (T - i)\theta_1$$

This is the human capital accumulation equation used to estimate α and the pattern of production elasticities, γ_i, over time.

[13]An additional reason for choosing a low order polynomial is possible correlation among resources over time. Amemita and Morimune (1974) recently concluded that this is an appropriate procedure when using the Almon distributed lag.

[14]The full set of results is obtainable from the authors.

[15]Again the full set of results is available from the authors. Disaggregation of the full sample on the basis of level of pre-school capital stock resulted in findings similar to those obtained by disaggregating by track.

[16]Ideally, an absolute measure of cognitive knowledge would be used as a measure of growth in the stock of human capital. The relationship between the ideal measure and the percentile measure actually used can be shown mathematically by employing two simplifying assumptions. First, assume raw scores (y) on tests are normally distributed so the transformation of raw scores into standard scores (z) with a zero mean and a standard deviation of one is a linear one:

$$z_{it} = \frac{y_{it} - \mu_t}{\sigma_t}$$

where μ_t and σ_t represent the mean and standard deviation of the population

on which the scores were standardized for grade t. Second, assume the standard scores are distributed sufficiently close to the zero mean such that the transformation between standard scores and percentile scores (k_{it}) is approximately linear:

$$k_{it} = 50 + 50\left(\frac{y_{it} - \mu_t}{t}\right).$$

If the true model using raw scores is represented by Eq. (4), the model which should be estimated using percentile scores is:

$$k_{i,t+1} = 50A\,\frac{\sigma_t^{\alpha}}{\sigma_{t+1}}\left[.02K_{it} + \frac{\mu_t}{\sigma_t} - 1\right]^{\alpha} Z^{\beta}\; X_t^{\gamma_t} - \frac{\mu_{t+1}}{\sigma_{t+1}} + 1\ .$$

This model cannot be estimated using available data for no information exists on μ and σ over time. An experiment was conducted to determine the direction of bias in the pattern of γ_t coefficients if μ_t and σ_t increase with t. However, the results were inconclusive.

[17]We have assumed a zero rate of discount (r) because (i) there is no single correct value for r and (ii) a non-zero rate of discount does not change the general policy implications of this paper. Assuming a positive rate of discount, B becomes the present value of the stream of per pupil revenue to be received by the district over the school-life of the child. If K_T is maximized subject to the budget constraint $B \geq \Sigma\,[X_t / (1+r)^t]$ the resulting first-order conditions for optimal resource allocation over time are

$$\frac{\gamma_1'\,(1+r)}{X_1} = \frac{\gamma_2'\,(1+r)^2}{X_2} = \ldots = \frac{\gamma_T'\,(1+r)^T}{X_T}$$

In other words, the general conclusion of this paper that the rate of increase

in X_t should be the same as the rate of increase in γ_t' is altered to read that the rate of increase in X_t should be larger than the rate of increase in γ_t'. Even if γ' is constant over time, the policy implication is that the level of expenditures should increase over time.

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TABLE I

Estimated Structures of A Human Capital Accumulation Model For Q_1

Variable	Mean†	Coefficients††
Constant		-4.294
Percentile Score on Grade One I.Q. (K_0)	55.24 (9.41)	.631* (.056)
Cultural Index	4.68 (2.06)	.016* (.008)
Home Ownership	.682 (.47)	.049* (.022)
Family Size	2.345 (2.06)	-.028* (.007)
Years of Education of Mother	11.97 (1.89)	.085 (.074)
Years of Education of Father	11.86 (2.02)	.1[illegible]* (.069)
Proportion of Peers of Low Economic Status, Grades 1 - 8	.342 (.185)	-.060* (.012)
Purchased Inputs per Pupil Grades 1 - 8†††	2350.80 (200.62)	
V_1		.198* (.034)
V_2		-.026* (.007)
Standard Error		.247
R^2		.407
Number of Observations	669	669

* Statistically significant at .05 level, two-tail test.
† Standard deviation in parentheses.
†† Standard error in parentheses.
††† V_1 and V_2 represent the expressions, πx_i and $\pi x_i^{(T-i)}$, respectively; the time pattern of elasticities is computed from the exponents on V_1 and V_2.

TABLE II

Estimated Elasticities of Q_1 with Respect to Purchased Inputs, by Race and Track in School, and for Alternative Capital Stock Measures, Grades 1 - 8

	Q_1					Q_2*	Q_3*
		Race		Track			
Grade	Full Sample*	Blacks	Whites*	College Prep*	Business-Vocational		
1	.018	.132	-.120	-.040	.012	.071	-.050
2	.056	.132	-.034	.007	.021	.096	-.007
3	.090	.131	.040	.046	.027	.125	.043
4	.119	.129	.104	.078	.032	.157	.099
5	.144	.127	.159	.104	.036	.194	.164
6	.165	.124	.206	.125	.038	.234	.236
7	.183	.120	.245	.142	.039	.280	.318
8	.198	.116	.278	.154	.040	.330	.410
α	.936	.908	.930	.911	.886	1.069	1.069
$\bar{Q}_1$ †	46.59 (13.56)	39.97 (10.75)	54.11 (12.47)	59.73 (10.14)	40.49 (10.20)	33.87 (24.98)	47.50 (28.80)
Sample Size	669	356	313	212	457	669	669

* The coefficients are jointly significant at the .05 level, two-tail test.
† Mean and standard deviation of output measures.

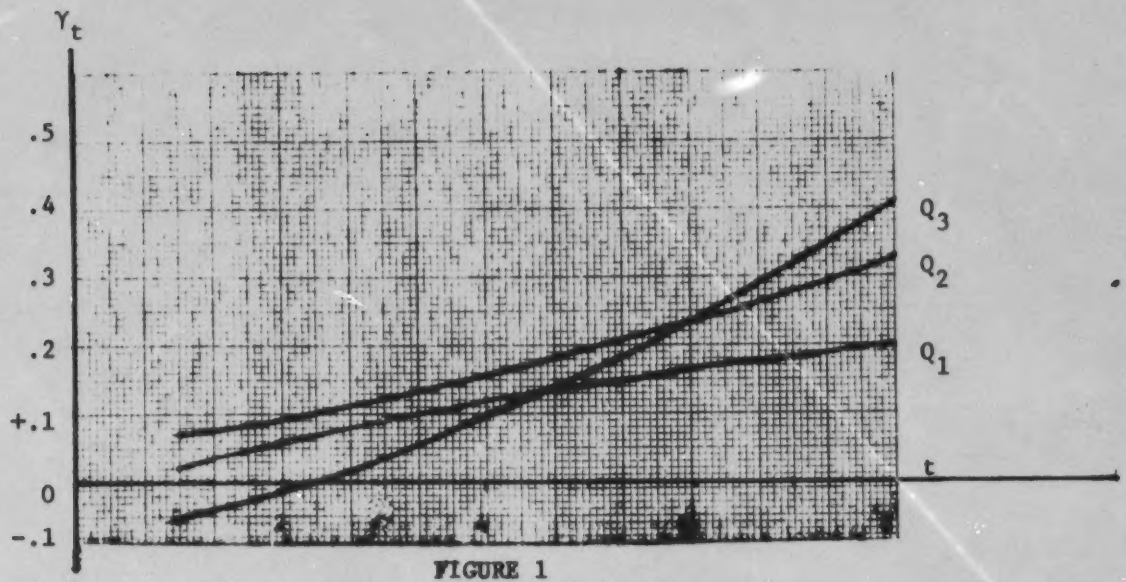

FIGURE 1

Estimated Purchased Input Elasticities of Production for Alternative Capital Stock Measures, the Full Sample, Grades 1 - 8.

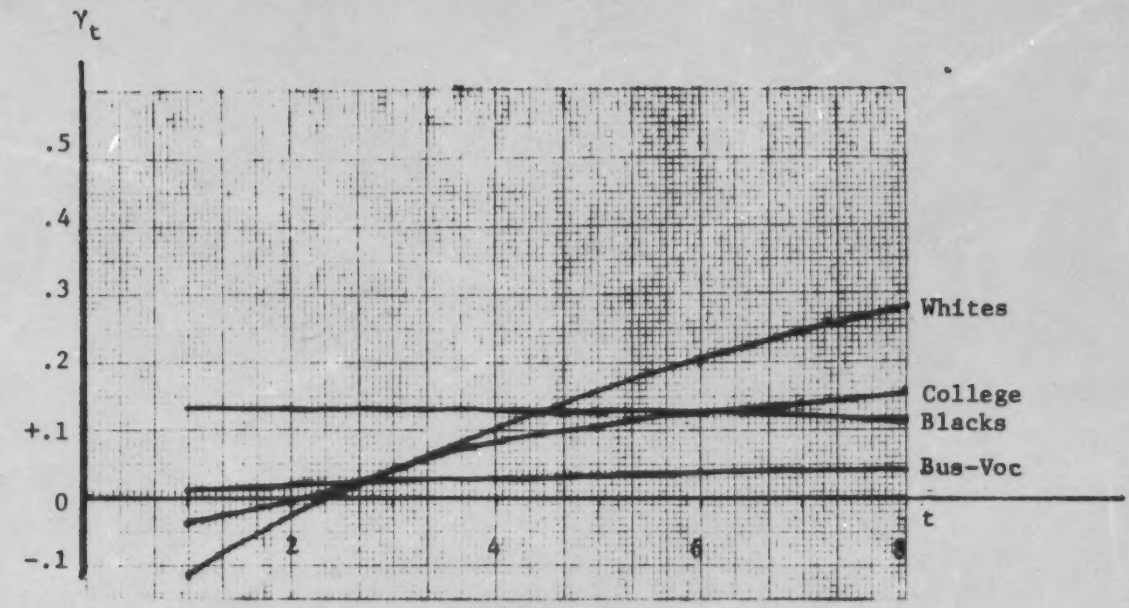

FIGURE 2

Estimated Purchased Input Elasticities of Production for Q_1, by Race and Track in School, Grades 1 - 8.

APPENDIX C

TEACHER PREFERENCES WITH RESPECT TO THE LEVEL AND DISTRIBUTION OF SCHOLASTIC ACHIEVEMENT*

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ABSTRACT

Education is a public sector activity which has been the focus of much economic research. Studies of educational production typically have assumed inputs to be exogenous and assumed pupils receive equal amounts of teacher time within the classroom. Here, an economic analysis is made of the effects of teachers on educational achievement under conditions where the teacher resources are distributed within the classroom according to a deterministic objective function. The maximization of achievement and the minimization of the variance in achievement are included as possible teacher objectives. The production function is assumed to be Cobb-Douglas. The results indicate the elasticities on teacher characteristics are small in size. Furthermore, teachers are found to strongly prefer maximization of average achievement to minimization of variance in achievement in the class.

The production and distribution of public output has received increasing attention from economists in recent years (Margolis, 1970). Elementary, secondary, and higher education represent a large part of total public sector activity and, perhaps for this reason, have been the focus of much economic analysis. Studies in publicly-provided education have analyzed the optimal provision of such public services (West, 1970), how such services should be financed (Reischauer and Hartman, 1973), the redistributive effects of public education (Hansen and Weisbrod, 1969; Kryzaniak and Eris, 1974), and the optimal allocation of resources within educational institutions (Razin and Campbell, 1972; Levin, 1970).

In order to draw intelligent conclusions on optimal resource allocation within schools or colleges, one needs information on both the prices of inputs and the precise nature of the production process. Levin (1974) and Bowles (1970) have pointed out some of the problems endemic to studies of educational production and, more generally, production of public outputs. The purpose of this paper is to investigate some further problems in estimating production functions of public outputs, again with specific reference to education.

Several well-known studies in educational production have found the effects of school resources on individual learning to be relatively small. (Coleman, 1966; Mosteller and Moynihan, 1972; Jencks, 1972). A common problem to many such studies has been that class average and not individual measures of school resources have been used as independent variables, thereby implicitly assuming that school resources are equally distributed across all students

within the classroom.[1] This is a convenient assumption, for average school resource measures are easily obtainable from school records, whereas individual resource measures would require an expensive and time consuming data collection. For example, measuring the allocation of teacher time within the classroom might require placing observers in classrooms. However, if the distribution of school resources over students is not equal and is related to any of the other independent variables in the regression equation, the estimated school effects are biased and may provide incorrect policy inferences.

In this paper we explore the effects of teacher inputs on individual achievement when the distribution of teacher time is simultaneously related to achievement. This relationship is viewed as determined by the teacher's objective function, which is assumed to include average level of achievement and variance in achievement in the classroom as arguments. The teacher resources used in the model are two measures of teacher quality: monthly salary and experience. Salary is, of course, in part determined by experience, but it is also determined by degree level and credits beyond the baccalaureate.

Recent research by Brown and Saks (1975) lend support to this type of objective function. Using aggregate data for the State of Michigan, they conclude that at least the distribution and mean level of student outputs should be included as arguments in the school's objective function. Using our data, it is not possible to determine whether or not the preferences of the schools with respect to level and distribution of outputs coincides with the preferences of teachers.

The results of the study indicate that teachers prefer a higher level of average achievement to a reduction in the variance of achievement. However, since the teacher effects are relatively small, the weights on the objective function of the teacher are a relatively unimportant factor in determining

individual student achievement. Although we were not able to do so, alternative specifications of teacher objective functions to take account of other school outputs such as socialization or discipline might lead to considerably different results.

The Model

Levin (1974) postulates a general formulation of the production function:

$$A_{it} = g[F_{i(t)}, X_{i(t)}, P_{i(t)}, O_{i(t)}, I_{it}] \qquad [1]$$

where:

A_{it} = a vector of educational outcomes for the ith student at time t

$F_{i(t)}$ = a vector of individual and family background characteristics cumulative to time t

$X_{i(t)}$ = a vector of school inputs relevant to the ith student cumulative to t

$P_{i(t)}$ = a vector of peer or fellow student characteristics cumulative to t

$O_{i(t)}$ = a vector of other external influences relevant to the ith student cumulative to t

I_{it} = a vector of initial or innate endowments of the ith student at time t.

The model of production employed in this paper differs from Eq. [1] in that the achievement ratio is expressed as a function of prior achievement and home and school input levels in the same time period:

$$\frac{A_{it}}{A_{i,t-1}} = f[F_{it}, X_{it}, A_{i,t-1}] \qquad [2]$$

where t and t-1 represent the level of outputs or inputs at time t and t-1 respectively. School peer characteristics, p_{it}, are omitted from the model because the sample of students all come from the same classroom; hence,

average peer characteristics are almost identical for each individual student.[2] Other external influences, O_{it}, are omitted because we have no measures of such variables. Innate endowments, I_{it}, are not omitted but are part of the recursive model, for $A_{i,t-1}$ is a function of $I_{i,t-1}$ as shown in Eq. [1].

The functional form of [2] is assumed to be Cobb-Douglas; the exponential specification is preferable to a linear one both for its mathematical properties and for the type of substitutability it permits between inputs. Only one element of the vector A_{it} is used as an output measure--test scores on standardized reading achievement tests, which are labeled Z_{it}. The structural equation of educational production is then represented:

$$Z_{it} = A \cdot F_{1it}^{\beta_1} F_{2it}^{\beta_2} X_{1it}^{\beta_3} X_{2it}^{\beta_4} Z_{i,t-1}^{\beta_5} \qquad [3]$$

where F_{1it} and F_{2it} represent the two family background variables--number of cultural items in the home and size of family, and X_{1it} and X_{2it} represent two school environment variables--teacher salary and teacher experience. The number of variables included in the model is limited both by the available data and the small number of observations.[3] Non-teacher school inputs such as books and supplies or building quality are assumed to be equally distributed among all students in the classroom.

Simple estimation techniques cannot be immediately employed, as there are no individual observations on X_{1it} and X_{2it}. We have only aggregate classroom figures for teacher salary and experience. However, if we assume a process by which X_{1it} and X_{2it} are generated, it is possible to derive functions for X_{1it} and X_{2it} in terms of variables for which there are empirical observations. In particular, it is assumed that the teacher has a utility function containing two elements: the average and the variance of achievement for his or her class, which he or she seeks to maximize

subject to two constraints. Formally:

$$\text{Max:} \quad U = \frac{\alpha_1 \Sigma Z_i}{n} + \frac{\alpha_2 \Sigma (Z_i - \bar{Z})^2}{n} \qquad [4]$$

$$\text{Subject to:} \quad \sum_i X_{1it} = n \bar{X}_{1t} \qquad [5]$$

$$\sum_i X_{2it} = n \bar{X}_{2t} \qquad [6]$$

Equations [5] and [6] embody the constraint that the teacher has a given fixed endowment of teaching quality and time.

The parameters α_1 and α_2 are, respectively, the marginal utility to the teacher of increasing average achievement and increasing variance in achievement. The ratio α_1/α_2 then represents the marginal rate of substitution between the two arguments in the utility function. Forming the Langrangian, dropping the t subscript for convenience, and manipulating the first order conditions for maximization, the following functional form can be derived:

$$\frac{X_{1i}}{\bar{X}_1} = \frac{X_{2i}}{\bar{X}_2} = \frac{\alpha_1 Z_i + 2\alpha_2 Z_i (Z_i - \bar{Z})}{\alpha_1 \bar{Z} + 2\alpha_2 \text{ var } Z} \qquad [7]$$

The educational production function of Eq. [3] can be reformulated with Eq. [7] to be:

$$Z_i' = A' F_{1i}^{\beta_1'} F_{2i}^{\beta_2'} Z_{i,t-1}^{\beta_5'} \qquad [8]$$

where

$$Z_i' = Z_i \cdot \left(\frac{\alpha_1 \bar{Z} + 2\alpha_2 \text{ var } Z_i}{\alpha_1 + 2\alpha_2 (Z_i - \bar{Z})} \right)^{\frac{\beta_3'}{1-\beta_5'}} \qquad [9]$$

and

$$A' = \left[A \bar{X}_1^{\beta_3} \bar{X}_2^{\beta_4} \right]^{\frac{1}{1-\beta_3'}} \qquad [10]$$

while

$$\beta_3' = \beta_3 + \beta_4 \; ; \quad \beta_1' = \beta_1/(1-\beta_3') ; \quad \beta_2' = \beta_2/(1-\beta_3') \text{ and}$$

$$\beta_5' = \beta_5/(1-\beta_3'). \qquad [11]$$

As a result of adding Eqs. [4] to [6] to the educational production function, we no longer have full identification of all the coefficients which we would like to estimate in Eq. [8]. The Cobb-Douglas specification and the constraints imply in Eq. [7] that any student receives equal shares of both teacher quality characteristics, an intuitively appealing result since teachers allocate their time and cannot independently allocate their characteristics.

Estimation

By assuming values for α_1, α_2, and β_3', we can obtain ordinary least squares estimates of A', β_1', β_2' and β_5'. We selected that combination of α_1, α_2, and β_3' which fit the data on Z_i' best by yielding the smallest sum of squared residuals. The alternative R^2 statistic would not be appropriate in this example, as changes in the assigned values for α_1, α_2, and β_3' lead to changes in the value of the dependent variable Z_i'.

In undertaking this analysis we are interested in discovering (i) the signs on the parameters α_1 and α_2 and (ii) the marginal rate of substitution between mean achievement and variance in achievement, α_1/α_2. Label the marginal rate of substitution α'. We discovered early that the sum of squared residuals is minimized when $\alpha_1 > 0$ and $\alpha_2 < 0$. As might be expected, the marginal utility of increasing achievement is positive, and the marginal utility

of increasing variance is negative.

Since the signs on the parameters were easily determined, the problem became one of estimating the marginal rate of substitution, α' , between the two arguments in the utility function. Combinations of discrete values of α' on a scale of 10 (-100, -90, ..., etc.) and β_3' on the scale (.00, .025, .05, .1, .2, ..., .9) were used for the estimations of A', β_1', β_2' and β_5' . The values which are reported in Table I are those which minimized the sum of squared residuals.

The nature and quality of all regression estimates crucially depend on our knowledge and assumptions about the error term. In general, anything which affects the dependent variable, but which is not contained in the right-hand-side variables, finds its home in the error term. First, if there is a misspecification and variables are contained in error terms which are not random, then we will get biased estimates of the variances of the coefficents. Furthermore, if the omitted variable is contemporaneously correlated with any of the right hand variables, there will be biased estimation of the coefficients themselves. We assume that there is no problem of misspecification, a point to which we return later.

Second, there are those random factors, like bad health on the day of the exam, which affect achievement scores. In this regression, obvious problems arise from such random disturbances as one of the right-hand-side variables, $Z_{i,t-1}$, is subject to the same problems because it measures achievement in the previous year by precisely the same method. In essence, we have a problem of errors in the measurement of the dependent variable and of one independent variable through the use of standardized tests. In general, such an error in the measurement of an independent variable leads to biased estimates. In this case there is no <u>a priori</u> suggestion as to the

direction of the bias because there is no reason to suppose that good health or bad health should systematically affect a particular level of achievement.

A final factor may enter the error term if teachers fail to optimize. The effects of non-optimizing behavior can be seen by adding a multiplicative error term in Eq. [7]. These error terms compound the normal error term but do not bias the estimated coefficients β_1', β_2' and β_5' because the variable from which they emanate has become part of the new right hand variable with the rearrangement of the regression equation. In general, additional randomness of the error term reduces the power of our estimates. In this example, if the non-optimizing behavior is not completely random, it increases the possibility of marginally incorrect choices of α' and β_3' as well. Our results, particularly those for α' and β_3', do not claim marginal precision in the first place; so non-optimizing behavior should not present any great difficulties.

Data

The econometric analysis was made using data on students and teachers in an urban school district in California. The data collected include measures of student achievement on the standardized examinations required in the State of California, measures of the student's home environment obtained from questionnaires, and measures of teacher characteristics obtained from personnel records.[4] Student observations were deleted from the sample if any of the information used in the model was missing.

The model described was tested for two classrooms of students in grades three, six and eight.[5] The achievement variables are percentile scores on standardized verbal achievement tests for the current (Z_i) and previous grade levels ($Z_{i,t-1}$) of the student. Number of cultural items (F_{1i}) is an index of

family possessions which ranges in value from 0 to 7. Family size (F_{2i}) is the number of siblings living in the home of the student. Teacher characteristics are monthly salary (X_{1i}) and years of experience (X_{2i}). The choice of the particular teacher resource measures to be included in the model is not too important in the framework used, as long as they have a statistically significant impact on achievement. Earlier cross-sectional research on this body of data demonstrated that to be the case.[6]

Regression Results

The values of α' and β_3' for which the sum of the squared residuals is minimized in the estimation of Eq. [8] are listed in Table 1 together with the estimated coefficients, their t- values, the R^2 and the number of observations, for all of the classes and grades selected.

Deviations in either direction from the "best" value of β_3' were found to increase the sum of the squared residuals dramatically. The effect of variations in the value of the preference ratio was less dramatic. This is hardly surprising given the low valuation of β_3'. Since teachers make such a small contribution, their particular preference functions and consequent distributions of time over students have little impact on educational achievement.

In addition to the consistency in choice of values for α' and β_3' shown in the six equations, the resulting estimates of the coefficients A', β_1', β_2', and β_5' are broadly consistent. Finally we can note that most of the signs of the coefficients conform to expectations and their values are usually significant at a 95% level of confidence.

It should be recognized that the results for α' and β_3' are not precise and there is no way of testing the significance of those particular

TABLE 1

Preference Ratio, Elasticity of Teacher Resources, Estimated Coefficients and Regression Statistics for Grades Three, Six and Eight

Grade (class)	Estimated coefficients (t- statistics) A'	β_1'	β_2'	β_5'	R^2	Numbers of observations	Preference ratio α	Elasticity of teacher resources β_3'
3 #1	1.889* (2.35)	.153** (1.93)	-.196* (-2.70)	.553* (8.32)	.463	36	-100	.025
3 #2	1.530* (4.27)	.089** (2.13)	-.064 (-1.40)	.658* (10.36)	.860	23	-100	.025
6 #1	1.835** (2.00)	.086** (2.08)	-.066** (-1.84)	.698* (5.41)	.744	29	-1	.1
6 #2	-1.988* (-7.20)	-.022 (-.93)	-0.047** (-1.97)	1.650* (17.08)	.938	25	-60	.1
8 #1	1.114 (.49)	.045 (.72)	-.123** (-2.15)	.821* (4.67)	.482	33	-100	.05
8 #2	-1.139* (-3.18)	.529** (2.36)	-.149** (-2.07)	1.132* (4.80)	.745	28	-60	.1

* Significant at .01 level.

** Significant at .05 level.

valuations. This being acknowledged, those particular valuations do very crudely suggest the magnitude of importance of teacher inputs to educational production at the margin, and they also suggest the relative weights attached to average achievement and variance of achievement in the teachers' preference functions. From the "best" values of the marginal rate of substitution, it appears that teachers in fact favor an increase in average achievement far more than a decrease in the variance in achievement. For example, grade three teachers are willing to trade off an increase in variance of one hundred points for a one unit increase in mean achievement.

At the same time, the values of the marginal products of the teacher resources which were calculated are of the same order of magnitude as those found in some of the earlier studies mentioned in Section 1. Elasticities are also close to those found by Hanushek (1972), who regressed average school achievement on average school resources for a cross-section of schools, using a Cobb-Douglas specification.

Conclusions and Policy Implications

The results discussed above have policy implications only if we assume the model to be correct. We noted the possible bias in estimates resulting from errors in $Z_{i,t-1}$, the effects of non-optimizing behavior. But the real problems arise from the possibility of mis-specification of the production function. Unfortunately, it is precisely here that the theory of learning or educational production provides little guidance for any discussion. One cannot produce statistically precise results with clear interpretations in a theoretical vacuum.

It may be this ignorance concerning the learning process which has led to the low value of β_3'. For example, ignoring a factor which is negatively

correlated to teacher inputs would lead to such a result, even if the real elasticity of teacher inputs were large. Yet even if the simple framework exposed here were a true representation of the process, the results do not necessarily imply that schools are ineffective in producing educational outputs. Schools produce a multifarious assortment of outcomes including attitudes and social responses. It is then quite plausible that while teachers have little influence on the production of reading ability, continued educational expenditures are justified for other purposes.

Schools have been assigned several missions in our society ranging from imparting cognitive skills to babysitting to acting as a selection mechanism for the labor market. Our findings are consistent with the latter function. If teachers in fact have stronger preferences for raising the average level of achievement than for reducing variance in achievement, selection is made easier by emphasizing or increasing knowledge differences between students. Furthermore, a Cobb-Douglas production function of the type postulated here implies that teachers with such preferences will allocate more time or more resources to high achievers or the socially advantaged.[7]

More research is required before strong policy implications can be made from these results. Not only should more attention be paid to the development of a theory of learning to guide empirical work, but researchers should also study the production of the whole spectrum of school outputs and the tradeoffs which exist between them. The results of this study suggest not only that the contribution of teachers to educational achievement as measured by standardized examinations is relatively small, but also that teachers weigh maximizing achievement more strongly than reducing the variance in achievement in their preference functions. This conclusion is consistent with the view that schools act as selection or sorting mechanisms for society

at large but in no way conclusively proves that sorting is the primary function of the schools.

FOOTNOTES

[1]An alternative assumption is that school resources, especially teachers, are pure public goods within the classroom, a reasonable assumption if teachers spend all their time lecturing to the class as a whole.

[2]By looking only at variation in student achievement within the classroom we have avoided another simultaneous equations problem in production wherein the best qualified teachers choose the classrooms with highest average achievement. See Nerlove (1965) for a discussion of the problem in estimating production functions in general; Greenberg and McCall (1974) present evidence that the problem does exist in education.

[3]For example, no information is available on family income.

[4]The tests used as output measures are the Stanford Reading Achievement tests for the respective grades.

[5]Whereas pupils in grades three and six had a single school teacher for the year, pupils in grade eight had more than one teacher, although all pupils in a "class" had the same set of teachers. The preference ratio then obviously pertains to some set of eighth grade teachers rather than one teacher alone.

[6]See Winkler (1975).

[7]One property of the Cobb-Douglas function is positive cross derivatives for any two inputs, $\frac{\partial^2 Z}{\partial X_1 \partial X_2} > 0$, which implies a larger marginal product associated with increasing teacher time to a high achiever or high socio-economic student than a low achiever or low socio-economic student.

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APPENDIX D

STUDENT SUPPLY OF LAB R IN LEARNING*

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ABSTRACT

Children can be viewed as making work-leisure choices within the context of the learning process. By foregoing current leisure in or out of the classroom, children can increase their cognitive knowledge and thereby increase their future earned income. In this paper we derive a student labor supply function which is simultaneously estimated with a production function of cognitive knowledge. Students are found to increase their work effort if the returns from that work, here measured by the marginal productivity of student labor, increase. Students are also found to increase their work effort if exogenous school or home inputs are increased.

The received economics literature on educational production has largely ignored student labor as a variable of interest. The productivity of student labor in learning has never been estimated, nor has much actention been paid to the factors which determine the amount of student labor supplied to learning. This paper attempts to remedy these omissions.

Like adults, children in school can be viewed as making work-leisure choices which maximize their long-run utility functions. By foregoing current leisure, children can increase their cognitive knowledge and thereby increase their future earned income.

The hours of schooling available is usually exogenous to children. However, they can increase their supply of labor to education by spending a higher proportion of school time actively engaged in learning activities. Children can also increase their hours of work by studying more hours outside of school.

The role of student labor in the educational production process has been generally ignored in past research. Exceptions are those studies which use days of school attendance as a proxy for student labor. For example, Wiley (1973) claims that variation in days of attendance is important in explaining variation in achievement scores in the data of the report on Equality of Educational Opportunity (EEO). Empirical studies which ignore the student input to learning in effect end up estimating reduced form equations of the learning process.

This study attempts to specify and estimate a model of the educational production process where the student labor supply is endogenous. This model enables us to provide tentative answers to some important questions. For example, how do changes in school inputs available to the child affect his supply of

labor? Or, how does home production of cognitive skills in the pre-school years affect the child's supply of labor to the learning process during the school years?

THE MODEL

Students are assumed to maximize their utility subject to constraints on time and educational production. The utility function is assumed to have the arguments current consumption, C_0, future consumption, C_f, current leisure, L_0, and future leisure, L_f. However, for the purposes of this paper we assume current consumption is exogenously determined by parents. Due to child labor and compulsory education laws, children in general cannot tradeoff current leisure for current income. We also assume children expect to work full-time in the future and, hence, view future leisure as fixed at some amount $\bar{L}_f$.

Assuming C_0 and L_f are fixed at $\bar{C}_0$ and $\bar{L}_f$, the problem as seen by the student becomes one of allocating his total time available, T, between work in learning, W, and leisure, L_0, so as to maximize utility.

The child's precise choice of present leisure and future consumption depends on (i) the possibilities for transforming leisure into future income and (ii) the child's own preferences for leisure and income. We assume future consumption is some monotonically increasing function of cognitive knowledge such that the tradeoff between current leisure and future income can be represented by the tradeoff between current leisure and current cognitive knowledge (Y_0). Unfortunately, past research on the relationship between cognitive skills and income does not lend strong support to this assumption.[1] However, what is important is how the child views this relationship, and educational folklore tells the child education is the path to higher income and social mobility. The problem formally stated is:

$$\text{Maximize} \quad U(\bar{C}_o, C_f, L_o, \bar{L}_f) \tag{1}$$

$$\text{Subject to} \quad T = L_o + W$$

$$C_f = f(W, Z) \tag{2}$$

where Z is a vector of other variables which enter the human capital production function.

The tradeoff between leisure and future consumption is the marginal product of student labor in educational production. Assuming diminishing marginal productivity of labor, this relationship is depicted in Figure 1.

The student choice of leisure and current consumption is determined by the point at which the product transformation curve is tangent to the highest indifference curve. This occurs at point A where the child receives L' hours of leisure and C' units of future consumption. At point A the rate of transformation is equal to the rate of commodity substitution.[2]

In this study we impose the constraint that all students face the same production function for cognitive skills, although the rate of product transformation depicted in Figure 1 may vary between students. However, we assume students may have different utility functions such that pupils facing the same production function may choose different combinations of leisure and future consumption. For example, students A and B in Figure 1 choose different combinations of L_0 and C_f and have different revealed rates of time preference.[3]

Effects of Other Inputs on Labor Supply

One factor which may cause the rate of product transformation to vary between students is the amount of other school resources, X, which they receive. Changes in X alter the optimal choice of leisure and future consumption. An increase in X can be shown to increase cognitive skills, and thus future consumption, but the effect on work effort is indeterminate.

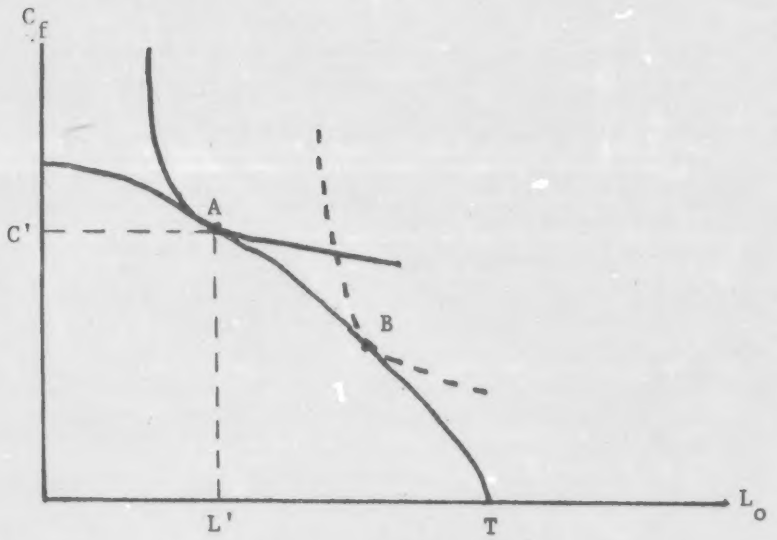

Figure 1

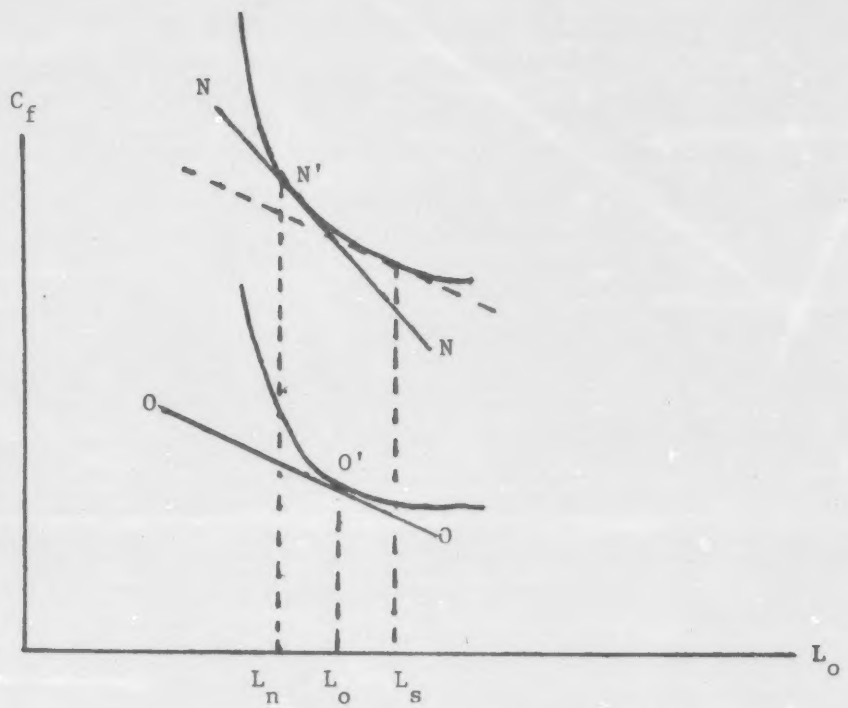

Figure 2

An increase in school resources, X, has offsetting income and substitution effects. In Figure 2 we depict a case where the marginal product of labor is constant over the range of values relevant to the pupil. Line 00 represents the original marginal product and point 0' represents the original choice of leisure and future consumption. Line NN represents the new marginal product after X has been increased, and N' is the new choice of leisure and future consumption. In this example work effort has increased from L_o to L_n. In other words, the substitution effect, L_s to L_n exceeds the income effect, L_o to L_s.

Since dW/dX is theoretically of an indeterminate sign, we later estimate a student labor supply function, which enables us to compute dW/dX. A student labor supply function can be derived from the first order conditions for the utility maximization problem [Eq. (1)] given above.

The family plays a large role in the production of cognitive skills, Y_p, in the preschool years. Like an increase in school resources, an increase in Y_p can be shown to have a positive effect on later cognitive knowledge but the effect on student labor supply is indeterminate. Hence, we estimate dW/dY_p within the context of the model.

SPECIFICATION OF THE MODEL

The Production Function

As is well recognized, there is little theoretical guidance for specification of the production function for cognitive knowledge. We assume, like Ben-Porath (1967), that the functional form is exponential or Cobb-Douglas.

The measure of output is, again like Ben-Porath, assumed to be the change in cognitive skills over a given period of time. In this case, we use as output, Y_o, the measured change in verbal skills between grades six and eight. The level of verbal skills is represented by the percentile scores of students

on standardized examinations of verbal skills. While a measure of the absolute level of cognitive skills of children would be desirable, no such testing instrument exists.[4]

(i) Home Inputs

The home and family are important in the production of cognitive skills both in the pre-school and the school years. The pre-school influence of the home is reflected in the level of cognitive skills of the child upon entering school, Y_p. Pre-school cognitive knowledge is a function of both genetic potential and family inputs. The two factors cannot be isolated.

The contribution of the home during the school years is here assumed to consist of parental time, parental human capital, and physical capital or material goods. Parental time is proxied by the number of siblings, S, living in the home. More siblings is presumed to leave less time for the parents to spend teaching any one child. Parental human capital is proxied by the number of years of education of the mother, E_m, and the father, E_f. Physical capital, K, is measured by an index of the number of items such as books, newspapers, etc. found in the home.

(ii) School Inputs

The school affects the cognitive skills of children by allocating resources to the students. The quantity of purchased inputs received by children is proxied by the average annual level of expenditures, X, on children in grades seven and eight. The quality of purchased inputs received by children is proxied by a measure of teacher quality, T. The variable T represents the proportion of a student's teachers from undergraduate institutions of higher education which required high scores on college entrance examinations for admission. Since the samples of students and teachers are from California, these schools are primarily the campuses of the University of California and Stanford

University.

(iii) Student Labor

Lastly, the student's own labor input, W, determines his learning. It would be desirable to have measures of how much student time within school and outside school is spent actively learning and studying. Unfortunately, such data is not available for this sample. Indeed, such data is rarely collected and never collected in a form which would allow estimation of the model posited here. Our measure of student labor is an ordinal index of the amount of time outside the school spent on learning activities. The scale of the input ranges from zero to five.[5]

The production function to be estimated thus takes the form:

$$Y_o = A\, Y_p^{\alpha_1}\, S^{\alpha_2}\, K^{\alpha_3}\, E_m^{\alpha_4}\, E_f^{\alpha_5}\, X^{\alpha_6}\, T^{\alpha_7}\, W_o^{\alpha_8} \qquad (3)$$

In terms of the above discussion we expect $\alpha_2 < 0$ and all other parameters greater than zero. The variables and their definitions are summarized in Table I.

Student Supply of Labor

The student's labor input is endogenous to the model posited here. In the typical labor supply model, the number of hours worked is a function of the reward received, usually the wage rate. In the student labor supply model, the number of hours worked is also a function of the reward received, but in this case the rewards are not necessarily expressed in money terms.

One of the rewards for the student is higher future income or consumption; this reward is proxied here by the marginal product of labor in the production of cognitive skills. Given the Cobb-Douglas production function [Eq. (3)] this reward can be expressed as $(\alpha_8 Y_0)/W_o$.

In addition to receiving a future, monetary reward for additions to the stock of cognitive knowledge, children receive current, non-monetary rewards from parents, and other pupils. The value the peer group places on academic achievement has been shown to be negatively correlated with the socio-economic level of the peer group [Wilson (1959)]. Hence, a child with predominantly low income peers could expect low peer rewards for improving achievement. Peer rewards are proxied here by the proportion of school peers of low social status, Z.

The family also rewards the child for academic achievement in non-monetary ways. The size of those rewards, we postulate, is positively related to the educational level of the parents. Teachers also reward pupil performance, but we have no measures of the types of rewards--letter grades and verbal communication--they are likely to give.

Differences in peer and parental rewards received for achievement by pupils is one reason why children facing similar marginal products of labor may choose different bundles of current leisure and future consumption as shown in Figure 1. Differences in the parameters in utility functions may also explain the phenomenon shown in Figure 1. We cannot distinguish between these two explanations. While highly educated parents may offer high rewards to children for academic achievement, they may also influence the parameters in the child's utility function.

Lastly, we hypothesize that the student supply of labor may be influenced by the income and wealth of his family. Increases in either variable might be expected to reduce work effort. As a surrogate for income, I, we use a dummy variable which takes the value e if the family was never on welfare and 1 if not. The surrogate for wealth, R, is a variable which takes the value e if the family owns its home and 1 if not.

Summarizing, the student labor supply equation is:

$$W_o = B\left[\alpha_8 \frac{Y_o}{W_o}\right]^{\gamma_1} Z^{\gamma_2} E_m^{\gamma_3} E_f^{\gamma_4} I^{\gamma_5} R^{\gamma_6} \qquad (4)$$

Rearranging terms, we have

$$W_o = B\,(\alpha_8)^{\frac{\gamma_1}{1+\gamma_1}} Y_o^{\frac{\gamma}{1+\gamma_1}} Z^{\frac{\gamma_2}{1+\gamma_1}} E_m^{\frac{\gamma_3}{1+\gamma_1}} E_f^{\frac{\gamma_4}{1+\gamma_1}} I^{\frac{\gamma_5}{1+\gamma_1}} R^{\frac{\gamma_6}{1+\gamma_1}} \qquad (5)$$

Eq. (5) is the equation estimated in the following section. In terms of the above discussion, we expect $\gamma_1, \gamma_3, \gamma_4 > 0$; $\gamma_2, \gamma_5, \gamma_6 < 0$.

SAMPLE AND DATA

The sample consists of 669 pupils who were enrolled in an urban school district in California in the mid-1960's. Since only those students with available school records from grade one through grade eight were selected, the sample is not necessarily representative of the school district as a whole.

The data on standardized test scores at grades one, six, and eight came from academic records of the pupils. The data on school inputs was also obtained by using academic records to match pupils to specific teachers and classrooms. Lastly, the data on home inputs and student work effort came from questionnaire responses of pupils.

The output measure, change in percentile scores on standardized achievement tests, is an imprecise measure of gain in cognitive knowledge. A zero change in percentile scores between grades six and eight does not imply the student gained no knowledge; rather, it indicates the student gained precisely the same amount of absolute knowledge as the sample of students used in standardizing the exam.[6] While average output as measured here is negative, real output in

TABLE I

Means and Standard Deviations of Variables

Variable	Mean†
Output Y_1	-15.10 (42.67)
Output Y_2	-23.34 (27.12)
Output Y_3	-31.99 (23.63)
Student Labor (W)	2.84 (.73)
Pre-school Knowledge (Y_p)	55.24 (9.41)
Number of Siblings (S)	2.35 (2.06)
Material Goods (K)	4.68 (2.06)
Mother's Education (E_m)	11.97 (1.89)
Father's Education (E_f)	11.86 (2.02)
Purchased School Inputs (X)	385.43 (33.76)
Teacher Quality (T)	.58 (.23)
Peer Group (Z)	.23 (.10)
Income Proxy (I)*	.74 (.43)
Wealth Proxy (R)*	.68) (.47)

† Standard deviation in parentheses.

* Dichotomous variables. The mean represents the proportion of the sample not on welfare (I) and owning their home (R).

terms of absolute knowledge gained is positive. Unfortunately, there is no known transformation between absolute knowledge and percentile test scores in different grades.

Because the output measure is imprecise, we test the robustness of our findings with respect to the particular output measure employed. Three alternative output measures are used.[7] The means of all output and input variables are given in Table I.

RESULTS

The model as represented by Eq.'s (3) and (4) has two endogenous variables in each equation. Hence, two stage least squares was used to estimate the structure of the model. The estimated structure of the production function is given in Table II; the estimated structure of the labor supply equation is given in Tables III and IV.

Production Function

(i) Home Inputs

The exponent on pre-school cognitive knowledge, Y_p, is positive and statistically significant in all three equations. The exponent on number of siblings, S, the proxy for parental time, is negative as expected and statistically significant in the first two equations. The estimated exponent on material goods or inputs, K, varies in sign and is never statistically significant.

Lastly, the elasticities associated with parental education, E_m and E_f, are always positive but not always statistically significant; furthermore, the values vary depending on the particular measure of output.

(ii) School Inputs

The production elasticity of purchased inputs is always large, positive,

TABLE II

Estimated Structure of the Production Function†

Variable	Output Y_1	Y_2	Y_3
Pre-school Knowledge (Y_p)	.63**	.89**	.34**
	(.22)	(.20)	(.15)
Number of Siblings (S)	-.13**	-.15**	-.03**
	(.06)	(.05)	(.04)
Material Goods (K)	.02	-.01	-.03
	(.06)	(.06)	(.05)
Mother's Education (E_m)	.37	.82**	.24
	(.28)	(.25)	(.20)
Father's Education (E_f)	.27	.47**	.42**
	(.27)	(.24)	(.19)
Purchased School Inputs (X)	.93**	.96**	.92**
	(.46)	(.41)	(.32)
Teacher Quality (T)	.19*	.21**	.09
	(.11)	(.10)	(.08)
Student Labor (W)	.24*	.29**	.25**
	(.13)	(.11)	(.09)
Constant	-9.22	-12.05	-8.44
Standard Error	.95	.85	.67

† Standard error in parentheses

* Statistically significant at the .10 level, two-tail test.

** Statistically significant at the .05 level, two-tail test.

and statistically significant. For each output measure, we cannot reject the null hypothesis that the elasticity has the value one, a surprisingly strong finding in light of the weak relationship between expenditures and learning usually reported in the literature. For example, in reanalyzing the EEO data, Hanushek (1972) found elasticities ranging from .04 to .22.

Teacher quality is found to be consistently positively related to learning, although the estimated exponenets are not always statistically significant. The measure of teacher quality used here is probably highly correlated with teacher verbal score, a variable which other studies [Coleman (1966), Hanushek (1972)] have found to be strongly related to student achievement.

(iii) Student Labor

Student labor input always exhibits a statistically significant, positive relationship to gains in cognitive skills. Furthermore, the estimated exponent is relatively stable with respect to changes in the measure of learning; the point estimates range from .24 to .29. *Ceteris paribus*, a ten percent increase in work effort on the part of the child is estimated to result in about a 2.5% increase in learning.

Student Supply of Labor

The elasticity of student labor with respect to the marginal product of his labor is always positive and statistically significant. The point estimate ranges in value from .25 to .37. The parameters associated with the other price variables, E_f, E_m, and Z , are not statistically significant in the two-stage least squares estimation, although some of those parameters were significant with the expected signs when the equation was estimated using ordinary least squares.

The exponents on family wealth, R , and income, I , are sometimes statistically significant at the .10 level. The exponent on wealth, as proxied

TABLE III

Estimated Parameters of the Labor Supply Equation [Eq. (5)]

Variable	Coefficients		
Output (Y_i)	(Y_1) .22** (.05)	(Y_2) .20** (.04)	(Y_3) .27** (.19)
Peer Group (Z)	-.01 (.02)	.01 (.02)	-.001 (.02)
Mother's Education (E_m)	.03 (.09)	-.05 (.10)	.05 (.09)
Father's Education (E_f)	.11 (.08)	.08 (.08)	.06 (.09)
Income Proxy (I)	.03 (.03)	.02 (.03)	.05** (.03)
Wealth Proxy (R)	-.05** (.02)	-.05* (.03)	-.02 (.02)
Constant	.57	.88	.67
Standard Error	.29	.29	.29

TABLE IV

Estimated Structure of the Labor Supply Equation [Eq. (4)]

Output	Variables					
	$(\alpha_8 \frac{Y}{W})$	Z	E_m	E_f	I	W
Y_1	.28	-.01	.04	.14	.04	-.06
Y_2	.25	.01	-.06	.10	.03	-.06
Y_3	.37	-.001	.07	.08	.07	-.03

by welfare status, is found to be positive.

DISCUSSION

We noted earlier that the effect of increasing school inputs or home inputs on student labor supply is theoretically indeterminate. An increase in some input exogenous to the student has a negative "income" effect and a positive "substitution" effect. The net effect can be either positive or negative. The estimated elasticities given in Tables II and IV enable us to predict the sign and size of these net effects.

The effect of an increase in school inputs, X, on student labor supply is $(dW/dY)\cdot(dY/dX)$ which, given Eq.'s (3) and (4), is equal to $(\gamma_1/1+\gamma_1)\cdot(\alpha_6 W/X)$. Since γ_1 and α_6 are both greater than zero, the net effect is positive as well. Similarly, we find an increase in home inputs increases student work effort. For example, an increase in pre-school cognitive knowledge, Y_p, changes work effort by the amoung $(\gamma_1/1+\gamma_1)(\alpha_1 W/Y_p)$, which is positive. In general, an increase in exogenous inputs in the production function has the net effect of increasing student work effort.

Because an increase in school inputs results in a positive change in labor supply, the estimated elasticity on X understates the total change in learning which results from a change in school inputs.

The total change in learning is:

$$\frac{\partial Y}{\partial X} + \frac{\partial Y}{\partial W}\frac{\partial W}{\partial X}$$

which is equal to

$$\frac{Y}{X}\alpha_6 + \left[\frac{\alpha_6\,\alpha_8\,\gamma_1\,Y}{1+\gamma_1}\right]$$

One of the surprising findings of this study is the statistically insignificant relationship between peer group composition and student work effort. Although the estimated elasticity was statistically significant in the ordinary least squares estimates, its size was very small (approximately -.04). A possible policy implication is that integration on the basis of social class can be expected to have little effect on the work effort, and hence the learning, of students unless other school inputs are simultaneously changed.

CONCLUSIONS

This paper represents a first effort at explicitly including student labor in the model of learning. While the estimated structures of the production and labor supply equations are theoretically reasonable, several problems which remain make policy inferences somewhat questionable. The major problems exist in terms of measurement of the endogenous variables. There exist no absolute measures of cognitive knowledge or changes in cognitive knowledge. Hence, we are constrained to use examinations which measure relative knowledge and which may change in content over time.

Student labor is also only roughly measured in this study. It would be desireable to have direct observations on student time spent studying in and out of the classroom instead of relying on students for the information. While precise labor data is conceptually possible to collect, the expense involved in making direct observations may be very large.

Other possible problems include the assumptions made about the arguments in the utility function, especially the assumption that gains in cognitive knowledge is a proxy for gains in future consumption. Future consumption is not solely determined by cognitive skills, and a richer model might explicitly take account of the other social factors and individual attributes which

determine income. A richer model might also permit students to tradeoff current leisure for current consumption. While our assumption that current consumption is fixed may be fairly accurate for students in elementary school, it is certainly less true for secondary school, especially high school.

Estimation of a richer model, however, awaits the construction of better measures of knowledge as well as the collection of better data. The collection of better data could begin soon with current studies on the use of time in the home. The home plays an important role in the production of knowledge. More precise measurement of parental time and student time spent in learning activities may generate information as to *how* the home affects student learning.

A number of psychologists [Carroll (1963), Block (1971)] are currently investigating the role of student time in learning in the classroom. Their research typically involves direct observation of the use of teacher and student time. As a complement to the studies of use of time in the home, economists might become involved in planning and analyzing the experiments being carried out by psychologists.[8]

This paper should be considered as a preliminary effort at modeling the role of student time in learning. We have found a positive, statistically significant elasticity of learning with respect to student labor. Furthermore, we have found a positive "price" elasticity of supply of student labor; the exponent on marginal productivity of labor is positive. Lastly, we conclude that increasing those school and home inputs which directly affect learning also influence the labor supply decision of the student. The substitution effect of increases in such inputs outweight the income effect of such increases; hence, an increase in some exogenous input results in an increase in student labor. An interesting policy question, which cannot be answered here, is what

allocation of resources in the school or allocation of parental time in the home would result in the largest student supply of labor.

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FOOTNOTES

[1]Recent research by Griliches and Mason (1972) and Hause (1972) has found the relationship between measures of cognitive skills and income to be small in size. The size of the relationship is larger, however, if one explicitly takes into account the effects of test scores in determining years of education attained by individuals [Ribich and Murphy (1975)].

[2]The marginal rate of substitution between current leisure and future income is equal to ρ times the marginal rate of substitution between current income and future income, which is usually labeled the individual's rate of time preference. Rho in turn is equal to the marginal rate of substitution between current income and current leisure. In the usual analysis, ρ is found to be equal to the wage rate. Since Parsons (1974) has estimated the wage rate of males to be approximately one dollar ($.99 to $1.25), the marginal rate of substitution between current leisure and future income is roughly equal to the rate of time preference.

[3]If capital markets operated perfectly, the observed rate of time preference would be equal for all students. However, capital markets do not operate perfectly; there is no market mechanism by which elementary school pupils can borrow against their future earnings.

[4]The measure of learning used here is further tainted by the fact that the precise skills and knowledge measured by standardized examinations changes over the school life of a child. Furthermore, the examinations used in this study are all verbal in emphasis. Thus, the results obtained here do not necessarily extend to other types of cognitive knowledge.

[5]The work variable, W, is an ordinal index which can take a maximum value of four. Since $W = T - L$, we arbitrarily set T equal to five and subtract an index of leisure which ranges in value from one to five. The leisure index is equal to 1.0 plus the sum of dichotomous student responses to the following questions:

1. I seldom or never finish my homework. (true = 1, false = 0)
2. I spend a lot of time caring for siblings. (true = 1, false = 0)
3. While attending school, I work for pay ten or more hours per week. (true = 1, false = 0)
4. On the average I spend two or more hours per day watching television. (true = 1, false = 0)

[6]The scores used in this paper were standardized on the basis of a national sample, not the school district or classroom of the pupil.

[7]The three alternative output measures are:

Y_1 = grade eight Stnaford Reading Test percentile score minus grade six Stanford Reading Test percentile score.

Y_2 = grade eight Stanford Reading Test percentile score minus grade six California Test of Basic Skills reading percentile score.

Y_3 = grade eight Stanford Language Skills Test percentile score minus grade six California Test of Basic Skills language percentils score.

Also, Y_p = grade one California Mental Maturity Test percentile score.

[8]Preliminary economic research along these lines has been undertaken by Garner (1973) and Christoffersson (1971).

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